This exam consists of three parts (I, II, and III) with equal weights.
Part I.

Long Analytical Question

**Saving and Growth.** This question explores the relationship between saving and growth for workers living in a small open economy. The interest factor is fixed at $R$ and labor income is exogenous and grows by the factor $G$ each period,

$$P_{t+1} = P_t G.$$  \hfill (1)

1. Prove that for an infinitely-lived consumer whose permanent labor income grows according to (1), the ratio of human wealth to permanent income is constant at

$$h_t = H_t/P_t = \frac{1}{1 - G/R}.$$  \hfill (2)

2. In class, we showed that if labor income is perfectly certain at $Y_t = P_t$, the consumption ratio will be

$$c_t = (b_t + h_t)\kappa,$$  \hfill (3)

where $b_t = B_t/P_t$ is the ratio of bank balances to permanent income. Explain why the marginal propensity to consume out of human wealth is the same as the MPC out of bank balances. (Note: “they are both multiplied by $\kappa$” is not an answer; I want economics, not math).

3. Explain why the human wealth effect of an increase in $G$ is negative and large in this model. (Use a numerical example to illustrate its size).

4. Now consider an economy like the one described above, but with uncertain labor income as in **Tractable**: With a probability $\Omega$ the consumer will become permanently unemployed, but if he keeps his job his labor income will grow by the factor

$$\Gamma = G/\Omega$$  \hfill (4)

Use the DBC

$$m^c_{t+1} = \mathcal{R}(m^c_t - c^*_t) + 1$$  \hfill (5)

where $\mathcal{R} \equiv R/\Gamma$ to prove that for a consumer at the target level of wealth $m^c$, the saving ratio in this economy will be approximately

$$s^c \approx \left(\frac{b^c}{m^c - 1}\right)\gamma$$  \hfill (6)

where $\gamma \equiv \Gamma - 1$. (Hint: The level of saving is total income (labor income plus interest income) minus consumption).
5. Tractable showed that under logarithmic utility, the target $b^c$ can be approximated by

$$b^c \approx \left( \frac{1}{(\gamma - r) + \vartheta(1 + (\gamma + \vartheta - r)/\Upsilon)} \right)$$

(7)

Use this formula to explain the effect of growth on target bank balances in this model. Explain why the size of the human wealth effect from growth depends on the values of the other parameters, and why the size of the effect is smaller in this model than in the perfect foresight model. (You may wish to choose specific numeric values of the parameters in order to illustrate your point).

6. Use (6) to discuss the steady-state relationship between saving and growth in this model. In particular, describe the competing influences and explain what determines whether the relationship is positive or negative.

7. Now consider the behavior of an employed individual who was at the target level of $m^e$ leading up to period $t$. In period $t$ there is an unanticipated increase in the growth rate. Show the dynamics of this individual’s bank balances and saving rate over time, under two different assumptions about parameter values: That the steady-state relationship is positive, and that it is negative.

8. Suppose the parameters are such that the steady-state relationship between saving and growth is positive. An econometrician proposes to estimate something called “the marginal propensity to consume out of wealth” by performing a regression of the form

$$s_t = \alpha_0 + \alpha_1 b^c_t$$

(8)

over a span of time that begins in period $t - i$ and ends at some date $t + j$. Explain how the estimated coefficient $\alpha_1$ will depend on the length of the data sample extending back before $t$ and forward beyond $T$. Under what circumstances, if any, would the econometrician obtain an estimated coefficient approximating the $\gamma$ of (6)? Is this coefficient appropriately described as a “marginal propensity to consume out of wealth”? Why or why not?

9. Now suppose we change the model so that unemployed consumers have a guaranteed minimum level of income (unemployment insurance) which they can borrow against. Explain why this change to the model permits the possibility of a target value of $b^e < 0$. Discuss how the relationship between saving and growth would change for a model in which target $b^e < 0$, both for the short-run and the long-run.
Medium Discussion Question

Capital Market Imperfections and the Fed. Over the period 2007-2008, the Federal Reserve has taken several unusual actions in response to developments in the capital markets, including orchestrating the takeover of Bear Sterns by JP Morgan, pleading to be a lender of last resort to investment banks, and joining with the Treasury in a plan for a government takeover of Fannie Mae and Freddie Mac if they should fail.

In the model presented in class on capital market imperfections, the following condition was presented:

\[ \gamma > 1 + r + A(c, r, W, \gamma) \]  \hspace{1cm} (9)

1. Explain this condition, and use that model to provide a variety of interpretations of either the reasons for the Fed's intervention or the reasons its actions might be expected to improve the functioning of capital markets.

2. Suppose the “right” diagnosis of the credit market disruptions is that it has been discovered that the cost of verification of financial contracts is higher than had been anticipated. Discuss what this model would predict about the consequences of such an increase in verification costs.

3. Give an intuitive explanation for why a decrease in interest rates might not be an effective response to financial market problems caused by financial market imperfections.
PART II

This Part has two questions with equal weights.

**Question A**

Consider the Solow growth model in continuous time. Let $s$ be the saving rate, $n$ the population growth rate, and $d$ the depreciation rate. The aggregate production function is $Y = K^\alpha L^\beta$, where $Y$ is output, $L$ is labor, and $K$ is the capital stock. Note there is no technological change.

1. Assume that $\beta = 1 - \alpha$. Suppose that $Y$ starts at some arbitrary level $Y_0$ at time $t=0$. What is the path of $Y$ over time from $t=0$ to $t=\infty$? Do your best to (i) provide a precise mathematical answer to the question (ii) show a graph of how $Y$ behaves over time and (iii) give an economic explanation for the results.

2. According to the model analyzed in (1), what factors might cause one economy to have higher output than another economy at a point in time, and what factors might cause living standards to be better in one economy than in another? What factors determine the changes over time in an economy’s output and living standards?

3. Of the answers in (2), which are consistent with evidence from real-world economies and which aren’t? Discuss.

4. How are the results in (1) different if $\beta < 1 - \alpha$? Here, a graph and clear economic explanation will suffice; a mathematical derivation is not essential.

**Question B**

Consider the following model of staggered price setting (all variables are in logs):

There is a large number of identical firms. Each firm adjusts its price every two periods. Half the firms adjust every even-numbered period, and half adjust every odd-numbered period. If a firm adjusts in period $t$, it sets a price for the current period, $x^1_t$, and a price for the next period, $x^2_t$. Each firm’s optimal price at $t$ is

$$x^*_t = p_t + vy_t, \quad v > 0,$$

where $p_t$ is the aggregate price level and $y_t$ is aggregate output. A firm sets $x^1_t$ equal to its optimal price in the current period.
and \( x_t^2 \) equal to its expected optimal price in the next period, that is, \( x_t^1 = x_t^* \) and \( x_t^2 = E_t x_{t+1}^* \). The aggregate price level is the average of all prices in effect in the current period. Half of these prices were set in the current period and half in the previous period, so \( p_t = (x_t^1 + x_t^2)/2 \). Output is given by

\[
y_t = m_t - p_t,
\]

where \( m \) is the money stock. \( m_t \) is stochastic.

(1) Discuss the assumed formula for \( x_t^* \). What is the economic intuition behind it? What microeconomic assumptions could be used to derive the formula? What determines whether the parameter \( \nu \) is large or small?

(2) When were models like the one in this question developed, and who developed them? What points were economists trying to make with the models? Whose ideas were they trying to refute? Explain.

(3) Suppose that \( m_t \) follows an AR-1 process: \( m_t = \rho m_{t-1} + \varepsilon_t \). The innovation \( \varepsilon \) has a variance \( \sigma^2 \). Derive a formula for the variance of output in terms of \( \nu \), \( \rho \), and \( \sigma^2 \).

(4) Provide economic explanations for the effects of \( \nu \), \( \rho \), and \( \sigma^2 \) on the variance of output.
Directions: There are four parts to this problem. Question 3.1 (all 5 parts) is mandatory and you should choose TWO of the remaining 3 questions. When you are requested to sketch an argument, the best answer would probably give both a sketch of the relevant math to demonstrate the point and clearly sketch the economic reasoning. If you cannot do both, either the math or the reasoning, if well done, will get substantial credit. The parts will be weighed equally.

3.1.[Mandatory]. Consumption and the term structure of interest rates.

3.1.1 Simple theories say the consumption choice at $t$ should satisfy the Euler equation,

$$ U'(c_t) = \beta E_t[U'(c_{t+1})(1 + R_{t,t+1})] $$

where $R_{t,t+1}$ is a return from $t$ to $t+1$. Give an explicit statement of a consumption problem giving rise to this Euler equation and interpret the economic meaning of this marginal condition. In this expression is $R$ stated in nominal or real terms?

3.1.2 Dividing both sides by $U'(c_t)$ allows us to write,

$$ 1 = E_t[m_{t,t+1}(1 + R_{t,t+1})] $$

where $m_{t,t+1}$ is defined by all the relevant terms from the equation above and is known as the marginal intertemporal rate of substitution.

Eqn. (1) should hold for the one-period return of any asset held in equilibrium by the agent. Write this expression for a risk-free (seen from time $t$) bond, $R^f_{t,t+1}$. Explain how we could measure the expected marginal intertemporal rate of substitution using this expression.

3.1.3 Write a second version of (1), this time for a risky one-period return. Using the expressions for the risky and risk-free bonds, demonstrate a rela-
tion between the expected relative returns on the two bonds and the covariance of the risky return with the marginal intertemporal rate of substitution. Interpret this relation.

(Note: remember $\text{cov}_t(a, b) = E_t[ab] - E_t[a]E_t[b]$.)

3.1.4 The expectations theory of the term structure states that the return from holding an $n$-period bond should be the same as the return from buying a one-period bond, reinvesting it at maturity in another one-period bond, and so forth for $n$ periods:

$$(1 + R_{n,t,t+n})^n = E_t \Pi_{i=0}^{n-1} (1 + R_{t+i,t+i+1})$$

(2)

where $\Pi$ signifies taking the product of the indexed terms, and $R_{n,t,t+n}$ is the annualized return from the $n$-period bond.

Explain the summary claim that the expectations theory states that the $n$-period rate should approximately be equal to a simple average of expected future one-period rates.

3.1.5 State the two standard assumptions under which the Euler-equation-based reasoning about interest rates is consistent with the expectations theory.

3.2 Fiscal policy.

3.2.1 Explain Ricardian equivalence and sketch a framework under which it would hold.

3.2.2 There is a long history of running simple regressions of interest rates on deficits in order to discover whether deficits lead to higher interest rates. In reality, people often find little relation between deficits and interest rates. Give some reasons why this might be so.

3.2.3 Suppose a government runs a deficit at time $t$. The deficit is unexpected at the outset and will be bond financed. The government credibly announces
a plan to pay off the bonds over time using proceeds from a conventional labor income tax. Sketch an argument under which this deficit would be contractionary in the short run (reducing output) AND an argument under which it would be expansionary in the short run.

3.3 Economic integration. Take a two-country world in which trade is forbidden (autarchy) and the economies are hit by both common shocks and idiosyncratic shocks. Then the countries open up to trade and become fully integrated.

3.3.1 Sketch an argument as to why output growth in the two countries might become more correlated after opening AND an argument about why they would become less correlated.

(Hints on question 3, you may wish to think about the effects of idiosyncratic demand and technology shocks and the changes gains from trade might induce in the structure of the economies.)

3.3.2 Sketch an argument about why we would probably expect consumption growth to become more correlated after opening.

3.3.3 A few years ago some analysts measured that consumption growth had become more correlated across countries and made the argument that this was bad for economic stability and welfare. For example, one argument was that if consumption were less correlated, when one country has a negative demand shock, it will tend to benefit from the fact that others are unlikely to also be having a negative shock. Assess this argument.
3.4 In macro there are many baseline theoretical results that do not seem to be descriptive of reality but regularly arise in our simple models. Explain at least 3 of the following:

- Friedman’s argument that optimal monetary policy would drive the nominal interest rate to zero.
- Modigliani-Miller theorem that the value of a firm is independent of the mix of bond and equity finance
- Lucas’s argument that business cycles have trivial welfare costs.
- Hall’s argument that consumption should follow a random walk
- Sargent and Wallace’s argument that only surprises (unexpected changes) in monetary policy should affect the economy