The Johns Hopkins University

Department of Economics

Macroeconomics Comprehensive Examination

June, 2009

This examination has three parts. You are required to answer all questions. Each part will be weighted equally. You have four hours to complete the examination. Budget your time accordingly. Your answers should be complete but concise. Think before you write.

NOTE: All three parts include both easier and harder questions. You should spend roughly equal time on all three parts. It is unwise to spend so much time on one part that you do not have time to do the easy questions in the other parts.
Part I.

Consider an economy where learning by doing occurs. Two key elements of this economy are:

\[ Y_t = \left[ K_t \right]^\alpha \left[ h_t L_t \right]^{1-\alpha} \quad 0 < \alpha < 1 \quad (1) \]

\[ h_t = \sigma K_t \quad \sigma > 0 \quad (2) \]

where \( K_t \) is the capital stock, \( h_t \) is human capital and \( L_t \) is labor. Equation (1) is the production function for the economy, and equation (2) describes the accumulation of human capital. Observe that human capital, \( h_t \), accumulates via the process of physical capital accumulation. Neither firms nor workers invest in human capital. Human capital accumulation is effectively "learning by doing" and is a by-product of the process of physical capital accumulation.

The questions below are designed to compare the social planning equilibrium with a decentralized competitive equilibrium.

A. Social Planning Equilibrium: Define \( C_t \) as consumption and \( I_t \) as gross investment. The social planner for this economy chooses time paths for consumption and capital accumulation to maximize:

\[ \int_0^\infty u(C_t)e^{-\beta t} dt \quad (3) \]

subject to (1), (2) and

\[ Y_t = C_t + I_t \quad (4) \]

\[ \dot{K}_t = I_t - \delta K_t \quad \quad K_0 = \bar{K} \quad (5) \]

\[ L_t = \bar{L} = 1 \quad (6) \]

1. Use optimal control theory to write down the optimality conditions for the social planner.

2. Assume that the utility function is
\[ u(C_t) = \ln(C_t) \quad (7) \]

What is the social planner’s calculation of the growth rate of consumption in this economy?

B. Competitive Equilibrium:

1. Household Problem: The representative household’s budget constraint:

\[ C_t + I_t + Z_t = W_t h_t + R_t B_t + r_t K_t \quad (8) \]

Together with (5) and

\[ \dot{B}_t = Z_t \quad (9) \]

where \( B_t \) is the stock of government bonds, \( Z_t \) is the accumulation of government bonds, and \( R_t \) is the return on government bonds. Note that the budget constraint assumes that the labor supply of a household is exogenous and set equal to unity, so that labor income is \( W_t h_t \). Answer the following questions regarding the household:

a. What is the arbitrage condition between risk-free bonds and capital? Explain carefully what it means.

b. Using the arbitrage condition, show that the budget constraint, (8), together with (5) and (9) implies the following accumulation constraint for the household

\[ C_t + \dot{A}_t = W_t h_t + (r_t - \delta) A_t \quad (10) \]

where \( A_t = K_t + B_t \) is gross assets held by the household.

c. Assume the household chooses time paths of consumption and the accumulation of assets to maximize

\[ \int_0^\infty u(C_t)e^{-\rho t} dt \quad (3) \]

subject to (10). Use optimal control theory to derive the optimality conditions for the household problem. Note that the household does not choose, \( h_t \), human capital.
d. Assume that the utility function is

\[ u(C_i) = \ln(C_i) \]  \hspace{1cm} (7)

What is the growth rate of consumption for the household?

2. Firm Problem: Assume that the representative firm chooses capital, \( K_i \), and labor, \( L_i \), to maximize profits:

\[ [K_i]^\alpha [h_i L_i]^{1-\alpha} - W_i h_i L_i - r_i K_i \]  \hspace{1cm} (11)

Write down the optimality conditions for the firm. Note that the firm does not choose, \( h_i \), human capital.

3. Competitive Equilibrium: Combine the household problem, the firm’s problem, labor market equilibrium (\( L_i = \bar{L} = 1 \)), and the definition of human capital to derive the decentralized competitive equilibrium for this economy. Calculate the growth rate of consumption for the competitive equilibrium.

C. Comparison:

Compare the growth rate of consumption from the social planning equilibrium with that from the decentralized competitive equilibrium. Are they the same? If not, how are they different? Explain carefully and in detail!

D. Government Policy:

The government proposes to subsidize the private cost of capital to firms. Specifically, it provides a subsidy, \( \tau \), to firms so that the private cost of capital to firms is now \( (1-\tau) \tau \). Assume the government pays for this subsidy by imposing lump-sum taxes, \( T \), on households. Answer the following questions:

1. Show how the competitive equilibrium and specifically the growth rate of consumption is affected by the subsidy.
2. What is the subsidy, \( \tau \), that the government should set so that the competitive equilibrium coincides with the social planning equilibrium? Is it zero? If not, what should it be?

3. If \( \tau > 0 \), what is the rationale for the positive subsidy? In other words, why should the government subsidize investment in this model?
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Macroeconomics Comprehensive Examination. Part II

We consider an economy populated by a continuum of identical atomistic individuals indexed by $i \in [0, 1]$. There is one homogeneous consumption good. Time is discrete, infinite $t = 0, 1, 2, ...$ and individuals are infinitely-lived. The utility of consumer $i$ at time $t$ is given by

$$U_t^i = E_t \left( \sum_{\tau=t}^{\infty} \beta^{\tau-t} u(C_t^i) \right),$$

where $C_t^i$ denotes individual $i$'s consumption at time $\tau$.

Each individual receives the same stochastic endowment $Y_t$ (the country's output) in period $t$. Like in Lucas' "tree model", the output is the payoff on an asset that can be traded between individuals. Individual $i$'s budget constraint at time $t$ is given by,

$$N_t^i p_t + C_t^i = N_{t-1}^i (Y_t + p_t),$$

where $N_{t-1}^i$ is the number of units of asset held by individual $i$ at the beginning of period $t$ and $p_t$ is the unitary price of the asset in period $t$. Note that the asset is traded in period $t$ after it has yielded period-$t$ payoff.

The aggregate number of units of asset is normalized to $N_t = 1$. In equilibrium each atomistic individual holds the same quantity of asset, thus we have,

$$\forall t, i \quad N_t^i = 1.$$

1. Show that the price of the tree satisfies the equation

$$P_t = \beta E_t \left( (Y_{t+1} + p_{t+1}) \frac{u'(Y_{t+1})}{u'(Y_t)} \right).$$

You need to explain precisely the steps by which you derive this equation in order to get credit (merely commenting on the equation will not do).

2. What is the price of the asset if output is constant (denoted by $\overline{Y}$)? Explain the intuition for any relationship you find between the price of the asset and the discount factor $\beta$.

3. Assume that from period $t$ onwards, output follows a perfect foresight path $(Y_\tau)_{\tau \geq t}$. By iterating forward on equation (1) derive an expression giving $P_t$ in terms of the whole path $(Y_\tau)_{\tau \geq t}$. Does an increase in future output raise or lower the price of the asset (in other terms, what is the sign of $\partial P_t/\partial Y_\tau$)?
Explain how the answer depends on the relative risk aversion of individuals (or if you prefer, on their elasticity of intertemporal substitution of consumption, which is the more natural concept in this context). If there are opposing effects, explain what they are, how they work, and under what conditions one is likely to dominate the other.

4. We assume that
   (i) utility is CRRA:
   \[ u(C) = \frac{C^{1-\gamma}}{1-\gamma}, \quad \gamma \geq 0, \quad \gamma \neq 1 \]  \( (2) \)
   and \( u(C) = \log(C) \) if \( \gamma = 1 \); and
   (ii) output grows by the constant factor \( G \geq 1 \) in every period, i.e.,
   \[ Y_t = G^t Y_0. \]
   We assume \( \beta G^{1-\gamma} < 1 \). Find a closed-form expression for the price of the asset. How does the price of the asset vary over time? Explain how the price of the asset varies with the growth rate. Why do we need to assume that \( \beta G^{1-\gamma} < 1 \)?

5. Assume now that \( Y_t \) is stochastic and identically and independently distributed (i.i.d.). Utility is still given by (2). We want to derive an expression for the price of the asset as a function of current output. Look for a function \( \tilde{P}(\cdot) \) such that \( P_t = \tilde{P}(Y_t) \) and show that it is given by
   \[ \tilde{P}(Y) = \frac{E(Y^\gamma)}{1 - \beta} Y^\gamma. \]  \( (3) \)
   where \( Y' \) is next-period output. Then derive a similar expression for the price of a "perpetuity", i.e., a bond that yields one unit of good in every period (you will denote it by \( \tilde{p}(Y) \)).

6. The assumptions of the previous question still apply. In addition we assume that output is given by,
   \[ Y_t = 1 + \varepsilon_t \]
   where \( \varepsilon_t \) is i.i.d. of mean zero, and variance \( \sigma^2 \). We assume that \( \varepsilon_t \) is small (first-order) so that one can apply second-order Taylor expansions \( (1 + \varepsilon)^\alpha \approx 1 + \alpha \varepsilon + \frac{\alpha(\alpha-1)}{2} \varepsilon^2 \). Derive expressions for the price of the asset, \( \tilde{P}(Y) \), and for the price of the perpetuity, \( \tilde{p}(Y) \), that involve the variance \( \sigma^2 \). Show that \( \tilde{P}(Y) \) is smaller than \( \tilde{p}(Y) \). Explain why the premium on the price of the perpetuity is a measure of the equity premium, and show that it is given by \( \gamma \sigma^2 \). Explain what the "equity premium puzzle" means, in the context of this model.
1. (a) Consider the IS-LM model.
   (i) What is the tax multiplier if the income elasticity of money demand is zero?
   (ii) What is the tax multiplier if the interest elasticity of money demand is zero?

   (b) Consider the Lucas imperfect information model. Let $V$ denote the ratio of the variance of aggregate shocks to the variance of idiosyncratic shocks.
   (i) Does a 1 percent surprise increase in the money supply have a larger effect on output if $V$ is big or $V$ is small?
   (ii) Does a 1 percent surprise increase in the money supply have a larger effect on inflation if $V$ is big or $V$ is small?

2. Consider the following simple dynamic model
   \[ \dot{y}(t) = -\kappa y(t) - \theta (r(t) - r^*) \]
   \[ \dot{\pi}(t) = \phi y(t) \]
   \[ r(t) = i(t) - \pi(t) \]
   where $y(t)$, $\pi(t)$ and $i(t)$ denote the output gap, inflation rate and nominal short-term interest rate, respectively, $r(t)$ is the real interest rate.

   The Fed sets the nominal short-term interest rate according to the rule
   \[ i(t) = \max(0, r^* + \pi(t) + \alpha(\pi(t) - \pi^*) + \beta y(t)) \]
   meaning that either the nominal interest rate is $r^* + \pi(t) + \alpha(\pi(t) - \pi^*) + \beta y(t)$ (the “Taylor rule”) or it is equal to zero. This is because the nominal interest rate can never go negative. The interpretation of $\pi^*$ is as the long-run “inflation target.”

   Assume that $\pi^*$, $r^*$, $\alpha$, $\beta$, $\kappa$, $\theta$ and $\phi$ are all positive constants.

   (a) Represent the dynamics of the output gap and inflation rate in a phase diagram. Mark clearly the direction that the system is moving at each point. Hint: the $\dot{y}(t) = 0$ locus contains a kink.
   (b) There are two steady state equilibria in this model. Find both equilibria, and state whether each of them is stable or not.
   (c) Suppose that the economy starts at $y(0) = 0$ and $\pi(0) = -r^* - 1$. Describe how this economy will evolve over time.
   (d) What are the policy implications of the result in (c)?