Macroeconomics Comp., June 2011

There are three major parts, labeled I, II, III.

You should attempt all three parts.

Partial answers and progress illustrating clear thinking will in many cases be as important as fully correct answers.

Best of luck...
Part I

Consider the following macroeconomic model designed to capture the interaction of investment, output, the price level, interest rates and equity prices (i.e., the stock market).

(1) \( \dot{Y}_i = \lambda (E_i - Y_i) \) \hspace{1cm} \lambda > 0 \hspace{1cm} Y_0 = \hat{Y}_0 \\
(2) \ E_i = C(Y_i) + I(q_i) \hspace{1cm} 0 < C' < 1 \hspace{1cm} I' > 0 \\
(3) \ i_i = \rho_i + \pi_i \\
(4) \ \frac{\dot{q}^*_{i}}{q^*_{i}} + \frac{X_{i}}{q^*_{i}} = \rho_i \\
(5) \ X_i = X(Y_i, v^*_i) \hspace{1cm} X_{\tau} > 0 \hspace{1cm} X_{\nu} > 0 \\
(6) \ i_i = \hat{i} + \phi_\pi \pi_i + \phi_\tau Y_i \hspace{1cm} \phi_\pi > 0 \hspace{1cm} \phi_\tau > 0 \\
(7) \ \pi_i = \frac{\dot{p}_i}{P_i} \\

where \( Y_i \) is real income, \( E_i \) is real private expenditure, \( q_i \) is the real value of equity prices, \( i_i \) is the nominal interest rate, \( \rho_i \) is the real interest rate, \( P_i \) is the price level, \( \pi_i \) is the rate of inflation, \( X_i \) is the level of real profits of business firms, and \( v_i \) is an index of optimism. The superscript “e” denotes an expectation, and a “dot” above a variable denotes differentiation with respect to time.

In working with the model, a number of simplifying assumptions will be used:

(A) Perfect foresight prevails.
(B) The price level is “sticky” so that \( P_i = 1 \).
(C) The index of optimism, \( v_i \), is constant so that \( v_i = \hat{v} \).
(D) The following condition is always satisfied: \( \phi_\tau > \frac{X_{L}}{q} \).
Answer the following questions:

(a) Provide a brief theoretical explanation (not just a description) of the following relationships:

(i) Provide an explanation of the relationship between the determination of investment, the equation that describes movement in equity prices, equation (4), and the equation that describes real profits, equation (5). If equation (4) were written in an "integrated" form, what would it say?

(ii) Provide an explanation of the interest rate rule, equation (6), and provide an explanation of Assumption (D).

(b) Next, impose the assumptions (A) - (D). Show that the model reduces to a system of two differential equations in \( q \) and \( Y' \). Which, if any, of these variables are state variables? Which, if any, of these variables are "jump" variables? Explain why.

(c) Then, analyze the dynamics of output and real equity prices. In particular, take a linear approximation of the relevant system of differential equations about its equilibrium. Determine the conditions under which the system of differential equation displays saddle point stability. In this regard, you may impose the assumption stated in (D). Draw a phase diagram to illustrate the dynamics of the model. Provide an economic interpretation of the dynamic adjustment of the economy to its equilibrium.

(d) Suppose the economy begins from an equilibrium at a full employment level of real output. Suppose then that a decline in optimism occurs. Specifically, suppose the index of optimism declines to \( v_1 = v \). Analyze the effects on investment, output, real equity prices, and real interest rates of a decline in the index of optimism. Specifically:

(i) Analyze first the effects of the decline in the index of optimism on the equilibrium values of real output, real equity prices, real interest rates and investment.

(ii) Then, analyze the dynamic adjustment of real output, real equity prices, real interest rates, and investment to their new equilibrium levels. Demonstrate your results with appropriate phase diagrams.

(iii) Provide a careful economic explanation of the dynamic adjustment of the economy to the new equilibrium. In particular, explain in detail how real equity prices, investment and output are affected by the decline in the index of optimism.
(e) Beginning from the new equilibrium, construct a monetary policy that will restore the level of real output to the full employment level. Analyze the effects on investment, output, real equity prices, and real interest rates of the relevant monetary policy. Specifically:

(i) Analyze the dynamic adjustment of real output, real equity prices, real interest rates and investment to the monetary policy. Demonstrate your results with phase diagrams. Provide an economic explanation.

(ii) When the economy reaches the full employment level of output, what will be the level of equity prices? Will it be higher or lower than the original level? In either case, explain why. What will be the effects on real interest rates and investment? Explain.
Part II.

Medium Analytical Question

**Consumption with Optimal Portfolio Choice.** CRRA-RateRisk shows that for a Merton (1969)-Samuelson (1969) consumer facing return $\log R_{t+1} \sim \mathcal{N}(r - \sigma_r^2/2, \sigma_r^2)$ on the only financial asset available, the optimal marginal propensity to consume is approximately

$$\kappa \approx r - \frac{1}{\rho} (r - \theta) - (\rho - 1) \left( \frac{\sigma_r^2}{2} \right)$$

(1)

1. Use this equation to discuss the parametric restriction(s) under which an increase in unavoidable financial risk $\sigma_r^2/2$ will cause a (precautionary) decline in consumption (the risk is unavoidable in the sense that we have assumed the risky asset is the only financial asset available). Explain whether, under such parameter value(s), the income effect of an increase in $r$ outweighs the substitution effect, and explain in words the intuition for both the income effect and the substitution effect.

For the remainder of the question, we assume that the consumer can choose how much to invest in the risky asset, financial risk is not unavoidable. **Portfolio-CRRA** derives the portfolio share $\zeta$ that an optimizing consumer will invest in a risky asset earning return $\log R_{t+1} \sim \mathcal{N}(r - \sigma_r^2/2, \sigma_r^2)$ so that $\log R = r$ (where the subscriptless version of a return factor denotes its expectation: $R \equiv E_t[R_{t+1}]$).\(^1\) The remaining proportion $(1 - \zeta)$ of the portfolio earns a riskless return $r = \log R_r$,\(^2\) and we write the expected return premium factor as $\Phi \equiv R/R$ with expected return premium $\phi \equiv \log \Phi = r - r = \log R/R$; optimal choice of $\zeta$ yields a portfolio whose realized return factor is written $\mathcal{R}$ (using the Gothic font)

$$\log \mathcal{R} \equiv \log E_t[\mathcal{R}_{t+1}] = \zeta \phi + r,$$

(2)

and Portfolio-CRRA shows that under these circumstances the optimal risky portfolio share is well approximated by

$$\zeta \approx \left( \frac{\phi}{\rho \sigma_r^2} \right).$$

(3)

and that the variance of the return on the optimally-chosen risky portfolio is approximately

$$\sigma_r^2 \approx \zeta^2 \sigma_r^2.$$

(4)

2. Show that the precautionary effect of rate-of-return risk on the precautionary contribution to the MPC, after taking account of optimal portfolio adjustment,

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\(^1\)Note the subtlety that the subscriptless version of a logarithmic return, e.g. $r$, is not equal to the mean: $E_t[r_{t+1}] = r - \sigma_r^2/2$.

\(^2\)The bold font is used for the risky return and the narrow font for the safe return.
is

\[- (\rho - 1) \left(\frac{(\phi/\rho)^2}{2\sigma_r^2}\right) \tag{5}\]

and explain the intuition for the result that the size of the precautionary effect
shrinks
as the risk grows larger.

3. Now use the foregoing results to show that the MPC \( \kappa \) can be rewritten in terms
of exogenous parameters (including the riskfree interest factor \( r \)) as

\[\kappa \approx r - \rho^{-1}(r - \vartheta) + (\rho - 1) \left(\frac{(\phi/\rho)^2}{2\sigma_r^2}\right) \tag{6}\]

and use this equation to discuss whether the total effect of an increase in risk on
consumption is positive or negative and explain why this result comes about.

4. Now show that the effect of an increase in risk on saving is given by

\[r - \kappa = \rho^{-1}(r - \vartheta) + \left(\frac{\rho + 1}{2}\right) \left(\frac{\phi^2}{\sigma_r^2\rho^2}\right) \tag{7}\]

and explain how this result relates to the results retained earlier. Comment, in
particular, on the role that our assumption that the consumer has no labor income
plays in obtaining the conclusion (hint: What further effect of interest rates is
omitted when the consumer has no labor income?)

II.2
Medium Discussion Question

Buffer Stock Savers with Sticky Expectations. This question asks you to combine some insights from the tractable buffer stock model and the sticky expectations model.

StickExpectationsC examined consumption dynamics in an economy populated by consumers who, if they had perfect information, would be Hall (1978) random walk consumers, but who instead have ‘sticky expectations.’ If the probability with which each consumer updates his information about the macroeconomy is \( \Pi \), and the only kinds of shocks that exist in this economy are transitory, that handout shows that aggregate consumption dynamics will be characterized by

\[
\Delta C_{t+1} = (1 - \Pi) R \Delta C_t + \Pi \hat{\Theta}_{t+1}\kappa_{\xi_{t+1}}
\]

(8)

where \( \hat{\Theta}_{t+1} = \Theta_{t+1} - 1 \) is the deviation of the aggregate transitory shock in period \( t + 1 \) from its mean value of zero. (Assume \( E_t[\Theta_{t+n}] = 1 \ \forall \ n > 0 \) where \( E_t \) is the rational expectation).

1. Consider an economy which had experienced no transitory or permanent shocks for a long time leading up to period \( t \) and then experienced a one-time positive shock to transitory income so that \( \hat{\Theta}_t > 0 \). Draw a diagram showing the dynamics of aggregate consumption leading up to \( t \) and for several periods thereafter.

2. Suppose that an econometrician studying this economy can directly observe the magnitude of the transitory and permanent shocks it experiences in real time. Show that this econometrician’s prediction for the change in income in period \( t \) will be \( E_t[\Delta Y_{t+1}] = -\hat{\Theta}_t \).

3. Suppose this econometrician were to estimate a Campbell-Mankiw model of aggregate consumption growth under the assumption that households' expectations of income growth matched the rational expectation:

\[
\Delta C_{t+1} = \alpha_0 + \alpha_1 E_t[\Delta Y_{t+1}]
\]

(9)

What would he obtain for the coefficient \( \alpha_1 \)? Explain why, and discuss what the econometrician would have to say about whether the economy exhibits ‘excess sensitivity’ and/or ‘excess smoothness’ relative to the random walk benchmark.

4. Now consider a different economy. This one is populated by buffer stock consumers, and it very occasionally exhibits a change in its growth rate. The econometrician observes this economy over a long time span, in the middle of which the economy exhibited a change in the growth rate of income. (For example, in the U.S. income grew faster in the period 1947-1973 than it has grown in the period 1973 to 2011). Suppose that in the earlier half of the sample consumers did not foresee that the
growth rate might change; they believed it was permanently at some level $\Gamma^A$, and suppose further that when the growth rate changes to $\Gamma^B < \Gamma^A$ partway through the sample, consumers see this immediately but perceive this change will be permanent. Draw diagrams showing the patterns of income growth and consumption growth over the entire sample, explain approximately what coefficient $\alpha_1$ the econometrician could expect to obtain in a regression like the one above if the sample size is long.

5. Now suppose that the buffer stock consumers who populate this economy have sticky expectations rather than rational expectations, and the economy is subject to transitory aggregate shocks like in the sticky expectations economy as well as permanent shocks to growth as above. Explain how the econometrician’s estimate of $\alpha_1$ is likely to depend on the sample size and the precision with which it is possible to measure permanent components versus transitory components of income growth.
Short Analytical Question

Risk Aversion of the Value Function. Consider an infinite-horizon consumption optimization problem under uncertainty for a consumer solving a time-separable intertemporal optimization problem of the form

\[ v(m_t) = \max_{c_t} u(c_t) + \beta E_t[v(m_{t+1})] \]  \hspace{1cm} (10)

where the utility function is \( u(s) = (1 - \rho) s^{1-\rho} \).

1. Show that relative risk aversion of the value function is

\[ \left( -\frac{v''(m_t)c_t}{v'(m_t)} \right) = \rho c'(m_t) \]  \hspace{1cm} (11)

2. Explain why this definition suggests that consumers with lower market resource ratios \( m_t \) can be expected to be willing to spend more of their budgets on actuarially fair insurance. Explain the intuition carefully.
References


Part III.

Do either part III.1 or III.2 and everyone does III.3.

Part III.1. [Do either this part or part III.2].

1.1 The average annual growth rate of real GDP in advanced economies since 1960 has been approximately what?

1.2 What phenomenon are macroeconomists speaking of when they refer to 'the Great Moderation'?

1.3 A 25 percent rise or fall in the exchange value between the currencies of two advanced economies is a rare event. True/False explain.

Part III.2. [Do either this part or part III.1].

2.1 Briefly discuss. Central banks should attempt to maintain a 'buffer' or 'stability cushion' between the short-term nominal policy interest rate and zero.

2.2 Explain in general terms what economists mean by the 'efficient markets hypothesis' as the term applies to financial markets.

2.3 True/False and briefly explain. In most economies over the last 20 years, the prices of most consumer goods have changed at about the same rate as the general level of CPI inflation.
Part III.3. [mandatory]

The following is a sketch of a model that has been used to explain how the economy might produce too many liquid assets leading to something a bit like the financial crisis. There are two special features of the model, one interpretational and one substantive. Interpretational: liquid assets are interpreted to be safe real assets. Substantive: households get utility directly from holding liquid assets (that is, safe real assets).

There are three sets of agents: households, banks, and something we will call an other financial institution (OFI). There are three dates, 0, 1, and 2. All household decisions are made at time 0. Financial institutions do some trades when information is revealed at time 1. Final consumption by households happens at time 2.

The household problem. Households have an endowment that they can either either consume at time 0, or invest in safe and/or risky bonds. Both types of bond pay off at time 2. The risky bond costs 1 unit of time zero consumption and pays a gross return in time 2 consumption of \( R_r \) (subscript 'r' for risky). The safe bond costs 1 unit of time zero consumption and pays a gross return of \( R_s \) ('s' for safe) at time 2.

At time zero, the household chooses \( c_0, B_s, \) and \( B_r \), to maximize

\[
U = c_0 + \beta E_0 c_2 + \gamma c_s
\]

subject to:

\[
Q = c_0 + B_r + B_s
\]

\[
c_2 = R_r B_r + R_s B_s
\]

\[
c_s = R_s B_s
\]

where \( Q \) is the time zero endowment, \( c_0 \) is time 0 consumption, \( c_2 \) is time 2 consumption, \( E_0 \) is the time zero expectations operator, and \( B_s \) and \( B_r \) are holdings of safe and risky bonds, respectively. As is more clear below, \( c_s \) is total safe claims to time 2 consumption purchased at

III.2
time zero; \( \gamma > 0 \) measures the extra utility the household gets from holding safe claims. Finally, \( 0 < \beta < 1 \) is a discount factor, and household holdings of each bond must be nonnegative.

1.1. **Question.** If the household holds the risky bond in equilibrium, what is the expected gross return on this bond, \( E_0 R_r \)?

1.2. **Question.** If the household holds the safe bond, what is the gross return? What condition is required for this gross return to be greater than 1? (Assume this condition henceforth.)

1.3. **Question.** Interpreting the safe bond as the *liquid bond*, then \( R_s/R_s \) can be viewed as a liquidity premium. What is the liquidity premium in equilibrium if both bonds are held?

1.4. Now we introduce the other agents. The banks. Perfectly competitive banks sell both types of bonds to the households. A bank that sells \( B_s \) and \( B_r \) has time zero proceeds of

\[
I = B_s + B_r
\]

Banks must invest all funds received, \( I \), in a risky technology. This is the real problem in this model, since banks will be financing risky investments by selling safe bonds.

The investment technology of banks. At time 1, it is revealed to all whether the return to this investment will be in the good state (probability \( p \)) or bad state (probability \( 1 - p \)).

In the good state, the gross return to the bank at time 2 from real investment of \( I \) at time 0 will be \( f(I) > I \); \( f \) is concave.

In the bad state, agents learn at time 1 that the *expected* time 2 return on the investment of \( I \) is \( \lambda I < I \). More specifically, in the bad state, when time 2 arrives, the investment will have a gross return of either
\( \lambda I/q \) (with probability \( q \)) or zero (with probability \( 1 - q \)).

Because a gross return of zero is possible, the bank could not, using only this technology, pay off any safe claims.

The OFI. At time 1, when there is still uncertainty about whether the bank projects will pay out \( \lambda I/q \) or zero, the banks can sell these risky assets to the OFI in return for safe claims to time 2 consumption. The rate of exchange on this time 1 financial market is \( v \) expected units of time 2 consumption for 1 safe unit of time 2 consumption, \( v > 1 \).

Note: the attempt to model the crisis is perhaps becoming clearer. In the bad state at time 1, banks are stuck with risky assets that have fallen in value; they are forced to sell these risky assets to the OFI, potentially at a very bad price, in order pay off their liquid liabilities to the households.

Define the share of initial bank investment financed by safe bonds:

\[
m = B_s/I,
\]

**Question.** What is the maximum share, \( m^* \) of \( I \) that banks could finance using safe bonds still allowing them to pay off the safe bonds by trading with the OFI?

(Hint: Banks could, at most, sell all their investments to the OFI in the bad state.)

1.5. More on the banks. Assume that the representative bank maximizes net proceeds in time 2 consumption—that is, it maximizes investment proceeds minus payouts on bonds, taking into account any trades with the OFI, and taking \( v \) as fixed.

**Question.** Write an expression for \( \Pi \), the bank's net proceeds at time 2 from any choice of \( B_s \) and \( B_r \), and assuming that the bank buys just enough safe claims from the OFI in the bad state to pay off the bank's
safe bonds.

1.6. Question. Under what condition on $R_s$, $R_r$, $p$ and $v$ will maximizing $\Pi$ as defined in (1.5) cause banks to push the share of safe bonds to the maximum $m^*$ defined in (1.4)?

(Hint: Do the maximization under the constraint $m \leq m^*$ and find when the constraint binds.)

1.7. Suppose that the OFI comes into time 1 with $W$ units in total of the consumption good. It can store the good costlessly so that 1 unit stored at time 1 becomes 1 unit of consumption at time 2. Alternatively, the OFI can invest some portion of its resources in a risky time 1 technology. This technology is such that if $X$ is invested in aggregate, then the expected gross return at time 2 is $g(X)$, where $g$ is concave. Zero is in the support of the gross return, however, so only if the OFI stores some of its resources can it sell the banks safe claims to time 2 consumption.

At time 1, the OFI maximizes net proceeds taking $v$ (defined above) as given. In the good state, the OFI simply invests all of $W$ for expected net proceeds of $g(W)$. In the bad state, the OFI sells $S$ claims to the banks and stores this much of the good, investing the rest. Net proceeds are $g(W - S) + vS$.

**Question:** Using the OFI's first order condition for maximization in the bad state, give an expression relating $v$ to the marginal product of investment in the time 1 technology.

1.8. Endogenizing $v$. In equilibrium, $S$—the resources stored by the OFI—will equal aggregate safe obligations of the banks. We discussed the representative bank above. Call $\bar{B}_s$ the aggregate of safe bonds sold by all banks. We have that in equilibrium $S = R_s \bar{B}_s$.

**Question.** What does a rise in $\bar{B}_s$ do to the equilibrium $v$? What does a rise in $\bar{B}_s$ do to quantity of investment in the time 1 technology in
the bad state? And to the marginal productivity of that investment?

1.9. Suppose that households own the OFI and the banks so that all proceeds are consumed by the households at time 2.

**Question.** Either prove or attempt to explain why this economy can exhibit equilibria in which the banks sell the households ‘too many liquid (safe) claims.’ That is, a social planner would have the banks sell fewer safe claims. [Hint: consider in particular the case when the share of safe bonds in bank financing is at the upper bound, $m^*$. Also consider whether the banks in the maximization in (1.5) take into account the effects discussed in (1.8).]

This problem was derived from, 'Monetary Policy as Financial-Stability Regulation,' by Jeremy C. Stein.