You have four hours for this exam.

There are three parts of the exam with equal weights (Parts I, II, and III).
Part I. Answer all questions.

Consider a model of economic growth that allows for the accumulation of both human capital and physical capital and where there are increasing returns to scale:

\[
Y_t = K_t^{\alpha} \left[ (1 - \mu) H_t \right]^\beta \quad 0 < \alpha < 1 \quad 0 < \beta < 1 \quad \alpha + \beta > 1
\]

\[
\dot{K}_t = s Y_t \quad 0 < s < 1
\]

\[
\dot{H}_t = \gamma \mu H_t \quad \gamma > 0 \quad 0 \leq \mu < 1
\]

where \(Y_t\) is output, \(K_t\) is physical capital, \(H_t\) is human, \(\mu\) is the fraction of human capital that is devoted to the accumulation of human capital, e.g., through education, \(1 - \mu\) is the fraction of human capital that is devoted to the production of output, and \(s\) is the saving rate. Note that \(s\) and \(\mu\) are assume to be constant.

1. Define \(g_t = \frac{\dot{K}_t}{K_t}\) as the growth rate of physical capital and \(z_t = \frac{\dot{H}_t}{H_t}\) as the growth rate of human capital. Then, derive the equations for \(g_t\) and \(z_t\) implied by the model.

2. Derive a differential equation that describes the rate of change of the growth rate of physical capital, that is, for \(\dot{g}_t\).

3. Define \(g^*\) as the steady state growth rate for physical capital and \(z^*\) as the steady state growth rate for human capital. Calculate both \(g^*\) and \(z^*\). Compare \(g^*\) with \(z^*\). Explain your results in economic terms.

4. Now, construct a phase diagram to illustrate the nature of the dynamics of the growth rate for physical capital. Specifically, given an initial growth rate for physical capital, \(g_0 = g_0^*\), construct a phase diagram that describes how the growth rate of physical capital, \(g_t\), adjusts to its steady state level, \(g^*\). Is the adjustment stable? Then, provide an economic explanation of how the growth rate of physical capital adjusts to its steady state level.

5. Suppose now that \(\mu\), the fraction of human capital that is devoted to the accumulation of human capital, increases. First, calculate the effect of the increase in \(\mu\) on the steady state value of the growth rate of physical capital, \(g^*\). Then, use
the phase diagram to describe the adjustment of the growth rate of physical capital to its new steady state. Explain your results in economic terms.

6. Define \( \lambda_t = \frac{\dot{Y}_t}{Y_t} \) as the growth rate of output. Compute the growth rate of output in the steady state. How does it compare with the growth rates of physical capital and human capital in the steady state? Explain your results in economic terms.

7. Next, consider the case where there are constant returns to scale so that \( \alpha + \beta = 1 \). Compare the results for the case of constant returns to scale with those for the case of increasing returns to scale. Specifically: (a) Compare the steady state values of \( g^* \) and \( z^* \). (b) Compare the phase diagrams that describe the nature of the dynamics for the growth rate of physical capital. (c) Compare the effects on the economy of an increase in \( \mu \) on the growth rate of physical capital. (d) Compare the effects on the steady state value of the growth rate of output. Most importantly, provide an economic explanation of any differences in results that may occur.
Part II.

Interpreting Global Saving and Asset Price Movements.

In an attempt to explain historically low levels of real interest rates, Bernanke (2005) hypothesized that a "global saving glut" had emerged.

1. Suppose we want to interpret the "saving glut" as reflecting a decrease in the time preference rate $\vartheta$ in a Ramsey-Cass-Koopmans model. Assume this decrease in time preference is permanent, and (a) show how the phase diagram changes; (b) show the dynamics of aggregate $c$, aggregate $k$, and the aggregate interest rate following the increase in patience. Explain why the curves and the variables move the way they do.

2. Suppose that, in the course of a single year, the higher saving rate associated with the glut results in the world capital stock increasing by 1 percent (a huge increase, by historical standards). Explain why, in a Ramsey model, if initially $k = \bar{k}$, the change in world interest rates associated with this increase in the world capital stock can be approximated by

$$\alpha(1.01\bar{k})^{a-1} - \alpha\bar{k}^{a-1} = \alpha\bar{k}^{a-1}(1.01^{a-1} - 1) \approx 0.01(\alpha - 1)\alpha\bar{k}^{a-1}$$

and, assuming an (empirically plausible) initial world target value of $\bar{k} = 8$ and $\alpha = 1/3$ show that the amount by which the world interest rate should fall is 1/18th of a percentage point.

3. Figure 1 shows the history of world real interest rates on riskless government securities since 1995. Using this figure and your insight from the prior question, explain why the benchmark Ramsey model cannot plausibly explain the decline in interest rates that started in 2001 as resulting from a "global saving glut."
Figure 1  World Real Interest Rates on Risk-Free Assets
4. Now suppose that we add a Hayashi (1982)-Abel (1981) type cost of capital adjustment to the Ramsey model; call the resulting hybrid the q-Ramsey model. Again assuming that the world economy was in equilibrium leading up to period $t$, draw the paths of real interest rates and of global asset prices that you would expect to see after a sudden increase in the global saving rate in the q-Ramsey model (on the same figure, show the corresponding patterns for the regular Ramsey model depicted above). **EXPLAIN the dynamics that you draw, in words.**

5. The q-Ramsey model does not make a distinction between the prices of different kinds of physical capital. Use the provided figures 2 along with whatever you know about global stock market returns and global house price changes in the 2000s to discuss whether the “global saving glut” seems to have affected all asset classes in qualitatively similar ways.

6. The world private saving rate increased somewhat further in the wake of the Great Recession that started in 2008; meanwhile interest rates on safe assets like government debt fell enough to hit new all-time lows (in the U.K., current nominal rates are the lowest since the first datapoint in 1694!). At the same time, the prices of risky assets (stocks, corporate debt, even housing) plummeted. What can you infer from these events about changes in the willingness of investors to hold assets considered risky? Are these data explainable solely by an intensification of the “global saving glut” Bernanke hypothesized?

7. Write down the Merton-Samuelson formula for the proportion $\zeta$ of a household’s portfolio that a household with relative risk aversion $\rho$ will choose to invest in risky assets. Use $\phi$ to signify the expected return premium on the risky asset (above the return on the riskless asset) and $\sigma^2$ for the variance of returns on the risky asset. Briefly explain the intuition for the role of each term in the equation.

8. Show that the standard model of unconstrained intertemporal choice under CRRA utility implies that the risk aversion of the value function is given by $\rho c'(m)$ where $c'(m)$ is the marginal propensity to consume and $m$ is the ratio of net worth to permanent income. Using this fact, explain how a large decline in a household’s wealth would lead that household to become more risk averse with respect to financial risks.

9. Suppose you had a microeconomic dataset containing complete household balance sheet data (that is, your dataset contains all data on income, consumption, and wealth for each household) over the course of the Great Recession for a representative sample of households. In this dataset, in addition to household wealth, suppose you also have good measures of the perception of household members about their degree of job security, and their perception of the degree of riskiness of
Figure 2 World Asset Prices

Chart 1

Global House Price Index
Flat or still falling?
global house price index, 2000=100)

(a) World Housing Prices

(b) World Stock Prices
investments in various kinds of assets. You can also see how individual households are allocating any new savings to risky and to safe forms of savings. How might you use such a dataset to distinguish between three theories:

a) The collapse in the prices of risky assets was caused in part by households' increased job insecurity, which led them to be unwilling to take other risks that might compound their nonfinancial uncertainty (the 'temperance' motive)

b) The collapse in the prices of risky assets was a 'self-fulfilling prophecy': As prices of such assets fell, the net worth $m$ of households owning such assets declined, resulting in a higher degree of risk aversion of the value function

c) Neither job insecurity nor self-fulfilling prophecies were involved; instead, households' perceptions of the underlying "true" degree of risk associated with different asset classes changed. Specifically, investments in housing, which had previously been viewed as safe, now became viewed as risky.
PART III

Classic Macro Models.

This question has two separate parts (A and B) with equal weights.

Part A

Consider the textbook Solow growth model. Let $s$ be the saving rate, $n$ the population growth rate, and $d$ the depreciation rate. Assume the production function is $y = k^a$, where $y$ is output per capita, $k$ is capital per capita, and $0 < a < 1$. Assume there is no technical change (unless a problem says otherwise).

(a) Suppose the economy starts in a steady state, and then one of the shocks specified below occurs. Draw graphs with time on the horizontal axis showing the paths of output per capita and consumption per capita from before the shock until the economy reaches a new steady state. Provide brief explanations, but no math or formal proofs are needed. If there is more than one possible answer, show all possibilities. Do this exercise for each of the following shocks:

i. There is a one-time, permanent increase in the saving rate $s$.

ii. There is a one-time, permanent improvement in technology: the production function becomes $y = \mu k^a$, $\mu > 1$.

iii. A large number of immigrants suddenly joins the population. (After that, population growth returns to its previous rate.)

(b) Now suppose a social planner chooses the saving rate for the economy, taking as given the other parameters of the model. The planner sets the saving rate to the value that maximizes steady-state consumption per capita. Suppose there is a permanent shift in the production function: As in the second shock above, output per capita changes from $k^a$ to $\mu k^a$, $\mu > 1$. What effect does this technology shift have on the planner’s choice of $s$? For this problem, prove your answer mathematically.

(c) In a paragraph or two, discuss the following statement: “The Solow model implies that higher saving rates raise per capita income. In the real world, countries with low saving rates tend to have lower incomes per capita than countries with high saving rates. Thus the textbook Solow model explains a large part of the differences in living standards across countries.”

Part B

Consider the following model of an economy:

Aggregate spending:

$$ y_t = a - br_t $$  \hspace{1cm} (1) $$

Money demand:

$$ m_t - p_t = c + dy_t - e_i $$  \hspace{1cm} (2) $$

Real interest rate definition:

$$ r_t = i_t - E_t(p_{t+1} - p_t), $$  \hspace{1cm} (3) $$
where $y$ is the log of real output, $r$ is the real interest rate, $m$ is the log of the money stock, $p$ is the log of the price level, and $\bar{r}$ is the nominal interest rate. The subscript $t$ indexes time periods, and $E_t$ is the expectation in period $t$. The parameters $a, b, c, d,$ and $e$ are positive. The variable $m$ is exogenous, while $y, r,$ and $\bar{r}$ are endogenous. We will consider different assumptions about how $p$ is determined.

(a) Draw the IS and LM curves for this economy. Label the axes carefully.

(b) Suppose $p_t=m_t=0$ for $t \leq 0$. For all $t \leq 0, p$ and $m$ are expected to stay at zero forever ($E_t[p_\tau] = E_t[m_\tau] = 0 \forall \tau$ for $t \leq 0$). However, a shock occurs in period 1: $m$ rises from zero to one. $m$ stays at one in periods 2, 3, ... $\infty$. Show graphically how the IS and LM curves shift from period 0 to period 1, and what happens to equilibrium output. Do this under three different assumptions about the price level:

i. $p_t=0$ for all $t$.
ii. $p_t=m_t$ for all $t$.
iii. $p_1=0$ and $p_t=m_t$ for $t > 1$.

For each case, assume that, when they make their decisions in in period 1, agents know the future paths of $p$ and $m$. Explain your reasoning in each case (but you don't need lots of math).

(c) Under which of the three assumptions about the price level does output change the most from period 0 to period 1? Under which assumption does it change the least? Explain.

(d) “Money is neutral if prices are flexible. The more rigid are prices, the greater the real effects of money.” Discuss this statement in a paragraph or two. Incorporate basic macroeconomic principles and the lessons of this problem.