Part I.

Theory and Evidence on the Effects of Credit Expansion.

A fascinating new paper by Aydin (2015) reports the results of an experiment in which consumers with credit cards received an exogenous increase in their credit limit. This question asks you to think about what results you would expect to find from such an experiment if all the consumers behaved according to a modified version of the TractableBufferStock model presented in class.

Our modification permits the model to analyze a change in credit availability. In order to explicitly incorporate a borrowing constraint that can be relaxed, the model must be changed in one respect: Rather than receiving income of zero, we will assume that unemployed consumers receive an ‘unemployment insurance’ (UI) benefit whose value in the first period (‘period 1’) of unemployment (the ‘replacement rate’) is $\mu$ times the value of their income in the last period of employment (‘period 0’); the benefit thereafter grows by the same factor $\Gamma$ as the aggregate wage.\footnote{This follows the approach elaborated in Carroll, Slacalek, and Sommer (2012).} To simplify the analysis, suppose that the UI program is financed by some windfall source of revenue (say, an oil discovery) and thus does not require an increase in taxes on employed consumers.

1. Explain the ‘growth impatience condition’ and show that, if it holds for the unemployed consumer, the PDV of UI benefits that an employed consumer will know he will receive if he becomes unemployed next period, relative to his current income, is:

$$ h = \left( \frac{\mu}{1 - \Gamma/R} \right) / R $$

where $\Gamma$ is the growth factor for aggregate wages, and $R$ is the interest factor.
2. Explain why, in a model with no exogenously imposed credit constraints, the creation of the UI program (an increase from $\mu = 0$ to $0 < \mu < 1$) implies that the consumer will be willing to borrow, but will never choose to allow his end-of-period cash-on-hand to fall to $-\bar{h}$ or below. Explain why the creation of the UI system can be interpreted as relaxing a 'natural borrowing constraint.'

3. Assume that before the creation of the UI system at date 1, the employed consumer was at the target level of market resources, $m_0 = \bar{m}_0$. (For convenience, henceforth please assume that the growth factor for aggregate wages is $\Gamma = 1$.)

   a) Show how the consumption function diagram changes for the employed consumer, and show a series of dots that trace out the immediate and eventual effects on consumption and market resources.

   b) Show that, if the MPC at the target level of wealth $\bar{c}_0$ is much greater than the interest rate (as is true for plausible calibrations of the model for impatient consumers), the infinite-horizon 'propensity to consume' out of the 'windfall' represented by the introduction of the UI program is (very) approximately $-r\bar{h}$. That is, show that, if $\bar{c}_0$ was the target level of consumption before the expansion and $\bar{c}_\infty$ is the new target toward which consumption tends, then $\bar{c}_\infty - \bar{c}_0 \approx -r\bar{h}$. Explain why eventually consumption is lower as a result of the relaxation of the constraint.

   c) Draw diagrams showing the time series paths for $c$ and $m$, and indicate the eventual new 'steady-state' levels of $c$ and $m$. How does concavity of the consumption function modify the results from what they would be if the consumption function were linear?

   d) Consider an increase in credit availability that occurs at date 0. Out of such an increase, define the 'marginal propensity to have consumed' (MPTHC) by the date $n > 0$ as the amount by which market wealth $m$ has declined divided by the amount by which credit was originally increased. That is, for a consumer whose wealth at date 0 was at its target value, $m_0 = \bar{m}$, the marginal propensity to have consumed by date $n$ would be $(\bar{m} - m_n)/\bar{h}$. Carroll (2001) shows that, in a model with a more realistic income process (transitory and permanent shocks calibrated to the PSID), a relaxation of an exogenous liquidity constraint results in a change in the target level of $m$ that is about the same size as the change in the availability of credit. In the simplified model considered here, the results in Carroll (2001) would correspond to the proposition that $\lim_{n \to \infty} m_0 - m_n \approx \bar{h}$. Under this assumption, draw a graph that shows the evolution of the MPTHC from date 0 toward date $\infty$.

4. Give some intuition for why the response to a relaxation of an 'artificial' borrowing constraint (such as a limitation imposed by lenders that requires the debt-to-income ratio to be smaller than a certain amount) is likely to be similar to the relaxation of the 'natural' borrowing constraint described above. (You can take this
to be true; your answer will be judged by the quality of your intuitive explanation for why).

5. Aydin (2015) defines the marginal propensity to consume out of liquidity as 'the dollar response of debt to a $1 change in borrowing capacity.' Given your answer to the question above about the MPTHC, critique this definition, particularly with respect to what it has to say about the time horizon.

6. Given his definition of the MPCL, Aydin (2015) makes the following statements about what his data show. For each statement, discuss the claimed result by deciding whether it is either (a) consistent with the results above from the tractable model; (b) inconsistent with the results above; or (c) ambiguous in whether it is consistent or inconsistent with the model.

   a) 'MPCL is significantly larger than zero for three quarters of the population, most of whom are not immediately constrained.'

   b) 'In the short-run, MPCL exhibits a clear heterogeneity in cash-on-hand, exclusively in line with a concave consumption function.'

   c) 'In the long-run, MPCL is mean-reverting: an increase in credit capacity does not permanently shift leverage, but individuals accumulate debt only to deaccumulate it after 6-18 months.²

The final part of this question invites you to contemplate consequences of the paper's findings for the analysis of the Great Recession. Assume for the purposes of the discussion that the paper's results suggest that there are two types of consumers: Highly impatient ones, with a low target buffer stock of assets; and mildly impatient ones whose buffer stock is very large. Mian and Sufi in a series of papers, and especially Mian, Rao, and Sufi (2013), have suggested that the credit boom that led up to 2007 largely reflected an increase in credit to the 'impatient' consumers who had small buffer stocks of assets, and that the post-2007 contraction in credit was concentrated on the same set of consumers. Recent work by Adelino, Schoar, and Severino (2015) challenges this proposition, arguing instead that much of the credit boom was directed toward consumers with substantial assets and income.

7. Draw the consumption function diagrams for highly impatient and for mildly impatient consumers, and use those diagrams to explain what the model says about why it might matter, for the consumption response, whether the credit boom was directed toward patient consumers (with initially large amounts of assets) or toward impatient consumers (with initially small amounts of assets).

²It must be noted that Aydin's evidence for this proposition seems weak. The figure cited as proof does not appear to me to exhibit any robust evidence of a tendency to deaccumulate. But, your task is to evaluate whether the proposition as stated is consistent with the model.
8. Some work has argued that another factor was involved in the consumption boom and subsequent bust: Uncertainty. Specifically, in the years leading up to 2007, macroeconomists were writing about the 'great moderation' in macroeconomic performance, the unemployment rate was stable at near-record low levels for many years, and direct measures of consumer uncertainty registered at low levels. After 2007, uncertainty by various measures spiked to near record high levels. Suppose that the degree of uncertainty faced by everyone is the same. Does the model imply that one group or the other (the 'highly impatient' or the 'mildly impatient') might react more to the increase in uncertainty? Sketch some implications for the research agenda.
References

ADELINO, MANUEL, ANTOINETTE SCHOR, AND FELIPE SEVERINO (2015): “Loan
Originations and Defaults in the Mortgage Crisis: The Role of the Middle Class,”


CARROLL, CHRISTOPHER D., JIRI SLACALEK, AND MARTIN
SOMMER (2012): “Dissecting Saving Dynamics: Measuring Wealth,
Precautionary, and Credit Effects,” Manuscript, Johns Hopkins University,
http://econ.jhu.edu/people/ccarroll/papers/cssUSSaving/.

MIAN, ATIF, KAMALESH RAO, AND AMIR SUFI (2013): “Household Balance Sheets,
1687-1726.
II.1) [20] Short Questions  
Answer the following questions in a few paragraphs each including some algebra:  
  a) [5] Describe the cyclical behavior of the debt service ratio and its relationship to the credit cycle. Explain the reasons for this behavior.  
  c) [5] Explain the intuition for each term of the equation $U = \frac{i}{i(1+i)}$ in a typical monetary model. Provide a model setup that results in the given equation as an optimality condition.  
  d) [5] Explain how rigid wages give rise to an upward-sloping aggregate supply curve in Keynes's model of the economy. Illustrate your explanation using a graph of the labor market.  

II.2) [25] Financial Imperfections and Interest Rates  
Formulate a simple two-period macroeconomic model with two sectors: (i) borrowers who face financial constraints as in Holmstrom-Tirole, and (ii) savers who have log period utility in period 0 and linear utility in period 1.  
  a) Describe the optimization problem of borrowers and derive the demand for borrowing.  
  b) Describe the optimization problem of savers and derive the supply of savings.  
  c) Describe equilibrium formally and in a graph.  
  d) Which structural parameter of the borrower problem governs the tightness of their financial constraint? How is the equilibrium interest rate affected when the financial constraint is relaxed? Describe formally and explain the economic intuition.  
  e) How is the equilibrium interest rate affected if borrowers have more initial wealth? Describe formally and explain the intuition.  

II.3) [30] A Simple Monetary Model Extended to an Open Economy  
Assume an open economy with a unit mass of identical domestic agents and two time periods, $t = 1, 2$. The utility function of a representative domestic agent is  
\[ U = \log(c_1) + \log(c_2) \]  
where the period 1 consumption good is an amalgamate of two goods, a traded and a non-traded good, combined using the Cobb-Douglas production function  
\[ c_1 = (c_T)^\gamma (c_N)^{1-\gamma} \]  
For simplicity, there are no non-traded goods in period 2 and $c_2$ just denotes the consumption of traded goods. The domestic agent obtains endowments of $y_T$, $y_N$ and $y_2$ of the period 1 traded and non-traded as well as of the period 2 consumption good. The consumption good in each period also serves as the numeraire good for its time period. Denote the prices of traded and non-traded goods in period 1 by $p_T$ and $p_N$.  
Domestic agents trade $b$ units of bonds with foreigners that are denominated in traded goods and that pay an exogenous interest rate of $r$, i.e. one bond costs
one unit of traded good in period 1 and repays \((1 + r)\) units of traded good in period 2. (The exogenous interest rate is a reasonable assumption as long as the country is a “small open economy,” i.e. it is small in comparison to world capital markets.) Furthermore, there is also a market for domestic bonds in which only domestic agents participate – they trade \(d\) units that are denominated in units of domestic consumption goods and that pay an interest rate \(i\).

a) Write down the optimization problem of domestic agents.

b) Given that there are two bonds, we obtain two Euler equations for domestic agents – one in terms of consumption goods, the other in terms of traded goods. Express the two Euler equations without shadow prices. (Hint: substitute for \(c_1 = (e_T)^\gamma (c_N)^{1-\gamma}\) and simplify; you may also want to use the Cobb-Douglas result \(p_T c_T = \gamma c_1\).)

c) State the arbitrage condition that ensures that domestic agents are indifferent between investing in the two bonds.

d) Use the Euler equation derived from foreign bond holdings to obtain an explicit expression for \(c_T\) and \(c_2\). Interpret how \(i\) and non-traded consumption \(c_N\) affects traded goods consumption in the described economy. (Hint: when expressing the intertemporal budget constraint of domestic agents, observe that domestic market clearing implies that \(y_N = c_N\) and \(d = 0\).)

e) Solve for the domestic interest rate \(i\) as a function of the exogenous parameters of the model. How does the domestic interest rate \(i\) depend on the foreign interest rate \(r\)?

The allocation described above represents the “flexible price equilibrium” in the economy. Now let us modify the setup and assume that the domestic central bank can determine the domestic interest rate \(i\) and that non-traded output \(y_N\) is demand-determined, subject to a maximum \(y_N \leq \tilde{y}_N\). In other words, \(y_N\) adjusts so as to satisfy the optimality conditions of domestic agents for a given domestic interest rate \(i\).

f) To what level does the central bank have to set \(i\) such that the maximum output \(y_N\) is obtained? Let’s call this level the natural interest rate \(i^*\).

g) Express the level of output \(y_N\) if the central bank raises the interest rate above \(i^*\).

h) [bonus] In the described economy, the relative price \(p_N/p_T\) can be interpreted as the economy’s real exchange rate. Obtain an expression for \(p_N/p_T\) as a function of the domestic interest rate \(i\). How does the real exchange rate respond to changes in the domestic interest rate \(i\)?
Part III. In part III, there are two questions and each has parts. All parts have equal value.

1. Note: Do 3 of 4.

(a) Aggregate investment shows no clear relation to the level of aggregate interest rates. Discuss several likely factors that may give rise to this outcome.

(b) Suppose that the central bank of an economy is empowered to freely purchase any foreign currency on a free market (for example, there are no capital controls). Starting from a given exchange value, and holding the behavior of all foreign central banks constant, this central bank can always lower the exchange value of its currency (depreciate its own currency) relative to any other single currency. True/False and explain.

(c) *Liquidity* is a term used frequently in economics, usually without clear definition. Discuss the meaning of liquidity as it is commonly used. (For example, you may consider issues such as: What features of the economy make liquidity an important concept? What determines the liquidity of an asset? What is a liquidity crisis?)

(d) Fama described one version of the efficient markets hypothesis as implying that ‘all public information’ is ‘fully reflected’ in market prices. He was, however, not very clear about the definitions of ‘all public information’ and ‘fully reflected.’ Discuss this notion of efficient markets and some of the complications or subtleties involved in formalizing this hypothesis. In practice, mathematicians and economists earn great rewards for proving theorems that hold under standard axioms. Is this a violation of the efficient markets hypothesis?
2. Do all parts. Take an economy in which there are a large number of identical consumers that each receive a quarterly endowment of the sole consumption good in the economy. The endowment has an aggregate component, $x_t$, and an idiosyncratic component, $z_t$. The income of the $i^{th}$ consumer is,

$$y_{it} = x_t + z_{it}, \quad t = 1, 2, 3, \ldots$$

where

$$x_t = 0.3x_{t-1} + \varepsilon_t$$
$$z_{it} = \alpha + \nu_{it}$$

where $\alpha$ is a constant and $\varepsilon_t$ and the $\nu_{it}$ shocks are mean zero, Gaussian, and mutually independent through time and across agents. The shocks $\varepsilon$ and $\nu_{it}$ have standard deviation $\sigma_\varepsilon$ and $\sigma_\nu$, respectively. To be clear, $\sigma_\nu$ is common across agents, $i$.

(a) Give an expression for aggregate income in this economy at any time $t$.

(b) Suppose that the consumption good can be stored across time periods, and that stored goods evolve such that $A_t$ stored at $t$ becomes $\delta A_t$ at $t + 1$.

Assuming perfect consumption insurance across agents in any period, what else do we need to know in order to derive an expression for each agent's optimal consumption at any point in time?

(c) Assume that $\sigma_\nu = 30\sigma_\varepsilon$. Suppose additionally that there is no consumption risk sharing at all, so that personal saving is the only way that agents can smooth consumption. Further, assume that agents separately observe $x_t$ and $z_{it}$ at $t$ and use this information in forming optimal consumption and saving decisions at $t$.

Making standard assumptions as needed, describe optimal individual consumption behavior in this case.

(d) Now suppose that agents never separately observe the two components of income, nor do they observe aggregate income. Indeed, the agents simply believe that their income has no correlation through time at all.

Describe consumption behavior that is optimal under these beliefs.
(e) Continuing from the previous part, suppose that a statistical agency in this economy could compute and inform all agents of aggregate income in each period before the consumers make their consumption and saving decisions; this informs the agents of the value of the separate component of income. The statistical agency has costs, however, that would be paid for by each agent paying a tax of $\tau y_t$ each period.

Discuss for what values of $\tau$ an optimizing consumer would be better off if the statistical agency exists. That is, what share of income would consumers rationally give up each period in order to be able to better smooth consumption in this economy?