You have four hours for this exam. There are three parts (I, II, and III) with equal weights.
Part I

There are debates about the economic costs of high government debt in advanced economies, as well as the implications of these costs for government debt sustainability. This problem set looks at these questions using a simple continuous-time intertemporal optimizing model.

There is one homogeneous consumption good that is produced by firms with labor. Firms use a linear production function,

$$ Y_t = A L_t, $$

where labor productivity, $A$, is assumed to be constant. The production sector is perfectly competitive. There is no money and all variables are expressed in real terms.

The economy is populated by a mass $I$ of identical atomistic consumers who maximize their intertemporal utility:

$$ U_0 = \int_0^{+\infty} \left( C_t - \frac{1}{2} L_t^2 \right) e^{-\rho t} dt, $$

where $C_t$ is the representative consumer's consumption at time $t$, and $L_t$ is his labor supply.

The budget constraints for the consumer and for the government are respectively given by:

$$ C_t + D_t = (1 - \tau) W_t L_t + r_t D_t, $$

$$ \dot{D}_t + \tau W_t L_t = r_t D_t, $$

where $D_t$ is government debt, which is held by the representative consumer, $\tau$ is the tax rate on labor income (assumed to be constant), $W_t$ is the wage, and $r_t$ is the interest rate. For simplicity we assume that the government does not have any expenditure except the interest payment on its debt.

1. What is the equilibrium level of the interest rate $r_t$? What is the equilibrium level of the wage $W_t$? In both cases, explain your answer in words and do not resort to long computations.

2. What is the elasticity of labor supply with respect to the wage? Show that the equilibrium level of labor supply, $L_t$, is constant over time and show how it depends on labor productivity and the tax rate on labor income. Is $L$ increasing or decreasing with the tax rate and why? How does the level of output per capita, $Y_t$, vary with $A$ and $\tau$?

3. Find a relationship between $D_0$ (the initial level of government debt), $A$ and $\tau$ that must hold in equilibrium. Show that if $D_0$ is larger than a critical threshold, there is no level of $\tau$ that satisfies this relationship. What do you think the government must do, then? [Hint: have a look at question 5.] If $D_0$
is lower than this threshold, is the tax rate \( \tau \) uniquely determined? If not, how do you think that the government will choose the tax rate, and why? Is the tax rate increasing or decreasing with \( D_0 \) and why?

4. Denote by \( d_0 = D_0/Y \) the initial government-debt-to-GDP ratio. Is GDP per capita \( Y \) increasing or decreasing with \( d_0 \) and why?

5. Assume that the government could default at time 0, in which case its debt is reduced to zero and the government does not need to levy any tax. However, a default disrupts the economy and permanently reduces labor productivity from \( A \) to \( A(1-\delta) \) where \( \delta \) is an exogenous parameter. The government decides whether or not to default so as to maximize the welfare of the representative consumer. What is the threshold in the debt-to-GDP ratio, \( d_0 \), that triggers a default? [You will first derive an expression that gives initial welfare, \( U_0 \), in function of \( \rho, A \) and \( \tau \).] What is the maximum level of the debt-to-GDP ratio that the government will bear without defaulting if \( \rho = 5\% \) and \( \delta = 1\% \)?
Part II.

Long Analytical Question

Buffer Stock Saving and Balance Sheets.

The Great Recession was particularly severe in economies that experienced a larger run-up in household debt prior to the crisis.

The International Monetary Fund (2012), Mian, Rao, and Sufi (2011), and Dynan (2012) have pointed out, respectively for countries, states within the U.S., and individual households, that those who ran up bigger debts in the period leading up to the Great Recession experienced bigger consumption drops in when the Recession hit. (The figure above shows some of the IMF’s evidence). This question asks you to interpret this pattern using a modified version of the tractable buffer stock model of saving, TractableBufferStock.

1. In the modified model, rather than having their income go to zero when they become unemployed, jobless persons instead receive an unemployment benefit
proportional to the labor income $\Delta W$ they earned in their last period of employment (call it period $-1$; the first period of unemployment is period $0$). Assume that these benefits are financed by some new source of revenues that does not affect the consumer’s budget constraint; foreigners, for example.

a) Call the beNefit $N_0 = \eta \ell_{-1} W_{-1}$, where $0 < \eta < 1$ means that the consumer’s retirement benefit is positive but less than the income they earned when employed. If in any period $t$ the consumer is ‘in debt’ $A_t < 0$ assume that a lender who does not receive interest payments of at least $-rA_t$ can seize any amount of the consumer’s income less than or equal to jobless benefits $N_0$ from the unemployed consumer. Explain why even an infinitely risk-averse private lender will be willing to lend an employed consumer any amount less than $H_{-1} = N_0/r$.

b) Explain the role of the ‘return impatience’ condition $(R\beta)^{1/p}/R < 1$ in guaranteeing that the solution to the unemployed consumer’s problem makes sense, in that a consumer who has ended employment with

$$A_{-1} > -H_{-1} = -N_0/r.$$  \hspace{1cm} (1)

will have strictly positive consumption throughout their unemployed life.

c) Explain why the existence of this unemployment insurance system is equivalent (in its implications for the path of consumption) to a system in which newly unemployed consumers receive a lump sum payment of $N_0(R/r)$ upon entering their first period of unemployment.

d) Call equivalent lump sum $\zeta \ell W$. Defining lower-case variables as the uppercase versions divided by $\ell W$, explain why $h_{-1} = \zeta = \eta/r$.

e) Explain why the effect of the introduction of such a system is simply to shift the consumption function in the phase diagram to the left by the amount $b = \eta/r$. Draw an example of such a leftward shift that is large enough so that the target ratio of bank balances $b$ is negative (so that in steady state the consumer will be in debt).

f) Explain why an expansion of unemployment benefits corresponds to a relaxation of a ‘natural borrowing constraint.’

2. Using the model, for each of the experiments below, show how the phase diagram changes, and show the path of the personal saving rate of debtors leading up to, during, and after the experiment.
Experiments:
a) Suddenly and without warning, consumers become more optimistic about the probability becoming unemployed: They believe there has been a permanent improvement in the functioning of the labor market so that the probability of unemployment $\mathcal{U}$ will be lower forever. This period of optimism lasts for six years, and then suddenly reverses itself (unemployment expectations revert to their previous value).

b) Suddenly and without warning, the government announces a permanent increase in the generosity of the unemployment insurance system (specifically: $\eta$ goes from $\bar{\eta}$ before period $t$ to $\bar{\eta} > \bar{\eta}$ in period $t$). Everyone believes this change is permanent. For six years, the change in the program persists. Then, without warning, the government reduces the generosity of the unemployment system back to its original level ($\eta$ goes back down to $\bar{\eta}$).

c) Suddenly and without warning, consumers become more optimistic about future income growth: They believe there has been a permanent improvement in $\Gamma$. This period of optimism lasts for six years, and then suddenly reverses itself (growth expectations revert to their previous value).
References


PART III

This part of the exam consists of two questions (A and B)

Question A

Consider a simplified version of the ISLM model:

\[
Y = C + I \\
C = C(Y), \quad 0 < C' < 1 \\
I = I(r), \quad I' < 0 \\
m^d = L(Y, i), \quad L_1 > 0, L_2 < 0 \\
m^d = m, \\
r = i - n^e,
\]

where \(Y\) is output, \(C\) is consumption, \(I\) is investment, \(m\) is real money balances, \(m^d\) is the demand for real balances, \(r\) is the real interest rate, \(i\) is the nominal interest rate, and \(n^e\) is expected inflation.

1. Assume that \(m\) and \(n^e\) are exogenous, and do the following:
   - Draw the IS and LM curves, carefully labeling the axes.
   - Suppose that \(m\) rises. Use the ISLM graph to show how this affects the equilibrium \(i\) and \(Y\), and give brief economic explanations for these effects.
   - Derive a formula for \(di/dm\), the effect of a unit increase in \(m\) on the nominal interest rate, and determine its sign. (The formula should involve the derivatives of various functions.)
   - Suppose that \(i\) begins at a level very close to zero. In this case, if \(m\) rises, are the effects on \(Y\) and \(i\) large or small? Justify your answer by showing what happens to the IS and LM curves in this case.

2. Continue to assume that \(m\) is exogenous, but let \(n^e\) be endogenous and determined by

\[
n^e = G(Y), \quad G' > 0
\]
• Give a brief economic justification for this new assumption.

• Draw versions of the IS and LM curves for this model, and explain clearly why they look similar to or different from the curves when \( \pi^* \) is exogenous.

• Is the sign of \( \frac{d\alpha}{dM} \) positive, negative or ambiguous? Give either a mathematical proof or a clear explanation using the ISLM graph.

Question B

Consider the following model of an economy:

\[
\begin{align*}
(i) \quad y &= -\beta r + \varepsilon, \quad \beta > 0 \\
(ii) \quad \pi &= \alpha y + \eta, \quad \alpha > 0
\end{align*}
\]

where \( y \) is the log of output, \( r \) is the real interest rate, \( \pi \) is inflation, and \( \varepsilon \) and \( \eta \) are independent shocks, each with a zero mean and a positive variance. Equation (i) can be interpreted as an IS equation and (ii) as a simple Phillips curve. Assume that \( r \) is chosen by the central bank.

1. Give interpretations of the shocks \( \varepsilon \) and \( \eta \). What kinds of economic events does each shock capture?

2. Compare the assumptions about the central bank’s policy instrument in this model and in the ISLM model. Which assumption is more realistic for modern economies? Explain.

3. Suppose the central bank sets the interest rate as a function of the model’s two shocks:

\[ r = \theta_1 \varepsilon + \theta_2 \eta \]

where \( \theta_1 \) and \( \theta_2 \) are coefficients chosen by the central bank. What values of \( \theta_1 \) and \( \theta_2 \) should the central bank choose in each of the following cases?

• Policymakers want to keep output stable.

• Policymakers want to keep inflation stable.

Show your reasoning.
4. Are the values of $\theta_1$ and $\theta_2$ that stabilize output the same or different from the values that stabilize inflation? Give economic explanations for your answers.

5. Suppose the central bank seeks to minimize

$$\text{Var}(y) + \text{Var}(n)$$

where $\text{Var}$ means variance.

- Is this a reasonable objective for the central bank? Explain.

- With this objective function, what values of $\theta_1$ and $\theta_2$ should the central bank choose?