You have four hours for this exam. There are three parts (I, II, and III) with equal weights.
Part I

There are debates about the economic costs of high government debt in advanced economies. This problem set looks at this issue using a simple continuous-time intertemporal optimizing model with *endogenous* growth.

There is one homogeneous consumption good that is produced and consumed by a mass of atomistic identical households. Households produce the good with capital using the linear production function,

\[ Y = AK, \]

where \( K \) is the capital held by the representative household and \( A \) is an exogenous constant. There is no money and all variables are expressed in real terms. There is no depreciation of capital.

The households maximize their intertemporal utility:

\[
U_0 = \int_0^{+\infty} \log C_t \cdot e^{-\rho t} \, dt,
\]

where \( C_t \) is the representative consumer's consumption at time \( t \).

The budget constraints for the consumer and for the government are respectively given by:

\[
C_t + \dot{K}_t + \dot{D}_t = (1 - \tau)AK_t + r_tD_t,
\]

\[
\dot{D}_t + \tau AK_t = r_tD_t,
\]

where \( D_t \) is government debt, which is held by the representative consumer, \( \tau \) is the tax rate on household income (\( \tau \) is assumed to be constant), and \( r_t \) is the real interest rate. For simplicity we assume that the government does not have any expenditure except the interest payment on its debt.

1. Derive the first-order conditions for the consumer’s optimization problem. You will show that the real interest rate is constant and equal to \( r = (1 - \tau)A \). You will also write the Euler equation relating the growth rate in consumption, \( \dot{C}_t/C_t \), to \( r \) and \( \rho \).

2. We consider in the rest of this exercise steady-growth paths in which all real quantities grow at the same rate \( g \),

\[
\dot{C}_t/C_t = \dot{K}_t/K_t = \dot{D}_t/D_t = g.
\]

(Take this as given, do not try to show it.) We denote by \( d \) the (constant) ratio of government debt to GDP. This ratio is an exogenous variable that is determined by the initial conditions, \( d = D_0/(AK_0) \).

Show that,

\[
\tau = (r - g)d.
\]
3. Using the expressions obtained in questions 1 and 2, find an expression that gives the growth rate $g$ in terms of $\rho$, $A$ and $d$. Is the growth rate increasing or decreasing with the debt-to-GDP ratio? Give an economic intuition for this result.

4. Assume that $\rho = 5\%$ and that in the absence of government debt the growth rate would be equal to 5% (i.e., $g = 5\%$ if $d = 0$). By how much is the growth rate reduced if the debt-to-GDP ratio is 100% instead of zero?

5. Show that the welfare of the consumer can be written,

$$U_0 = \frac{\log[(A - g)K_0]}{\rho} + \frac{g}{\rho^2}.$$

(You will use the identity $\int_0^{\infty} te^{-\rho t} dt = 1/\rho^2$.) Show that other things equal, a higher debt ratio decreases the consumer's welfare. Give the economic intuition behind this result.
Part II.

Several Short Questions

1. Open Source Software and Knowledge Capital. “Open Source” software is created largely by computer programmers who are not directly paid for their contributions. Some of the most central components of the infrastructure of the internet are open-source (for example, the Apache web server software that powers perhaps half of the servers on the internet; the FireFox browser; the LaTeX typographical system; and much more).

   Explaining the production of valuable services by volunteer labor is a challenge for neoclassical economics (to say the least!). This question asks you to make a first stab at the job, using the Lucas (1988) growth model. Recall that in that model, “human capital” \( h \) accumulates according to

   \[
   \dot{h}/h = \phi(1 - u)
   \]

   where \( u \) is the proportion of time that people spend not working.

   a) Write a paragraph or two about reasons for and against a reinterpretation of Lucas's model in which what he calls “human capital” is thought of as “knowledge capital” produced by people in their free-time contributions to open source knowledge projects. Comment, in particular, about what you need to assume about the parameter that captures the degree of externalities in aggregate \( h \) in order for this interpretation to have any force.

   b) Suppose a government concludes that the value of open source software in boosting aggregate productivity is very large. Under the appropriate assumptions (that is, the assumptions that make the Lucas model sketched above appropriate for addressing the question), suppose the government wants to evaluate a subsidy that will increase the leisure time of computer programmers (in the belief that in their leisure time they will create more open-source software).

   - Assume that only computer programmers contribute to knowledge capital in the relevant sense (everybody else just contributes raw labor). Also, programmers are “born that way” (they can't help becoming programmers, and nobody else can become one) so that their proportion in the population remains constant at \( \Pi \). Show that under these assumptions the effective aggregate production function becomes

   \[
   y = \dot{h}^{1/\alpha}(uh)^{1-\alpha}
   \]
where $h$ is the knowledge capital per capita of the computer programmers and $u$ is the proportion of their hours they spend working.

- Explain why the optimal policy for the government is likely to involve subsidizing the programmers not to work. Discuss how would the appropriate subsidy depends (qualitatively) on the different parameters in the model (including the Cobb-Douglas parameter for capital's share in output).

- Now suppose that labor market imperfections mean that the company that a programmer works for is able to temporarily capture a substantial portion of the “knowledge” benefits of the programmer’s “free time” programming activities, but the amount that the company captures depends on the number of hours that the programmer spends working at the company (the $u$). Will the company agree with the government about the optimal size of the subsidy? Explain why or why not, and discuss what determines whether the company is likely to benefit (on net) from the externalities mentioned above.

- Assuming that parameter values are such that the subsidy to leisure of programmers has a positive net value, would the model suggest that such a subsidy would have a positive effect on the level of output, on the growth rate of output, or both?

- How might such a subsidy compare, in its effects, to a more traditional policy like a permanent investment tax credit?

- Explain why the government's optimization problem becomes much harder if some people can pretend to be computer programmers even though their only real talent is in English Literature and they hate programming and in their free time they read Shakespeare instead of coding the latest update to Firefox.

2. Recent work by Mullainathan and Shafir (2013) can be interpreted as claiming that psychological effects of poverty (perhaps interpretable as "stress") effectively cause poor people to become more impatient. This question asks you to think about some implications of that hypothesis using a buffer-stock model of saving. Specifically, suppose that there are two psychological states that a person can be in: Either "relaxed" or "stressed." In the "relaxed" state the time preference rate is lower than in the "stressed" state. Suppose that, in either state, people do not perceive that there is a possibility that they might make a transition to the other state. That is, a person in the "stressed" state believes he will always stay in the stressed state, and similarly for the relaxed state. However, suppose that in truth, whenever a person's market resource ratio falls below a certain cutoff point, that person inevitably switches to the stressed state. If the $m$ ratio rises above the cutoff level (call it $\hat{m}$) the person will make a transition back to the "relaxed" state.
The reason "stress" has this effect is supposedly that the brain does not operate as well in a stressed condition as in a relaxed condition.

Answer the following questions under the assumption that a target wealth ratio exists for either a consumer in the "stressed" or in the "relaxed" state.

- Whose target wealth ratio is higher, the "stressed" person's or the "relaxed" person's? (Draw the phase diagram showing the target level of wealth for the two kinds of people).
- Suppose a "relaxed" person who is at the target level of wealth for a relaxed person suddenly gets hit by a negative shock to wealth (car crash; child's wedding; medical expenses) that push his wealth below $\bar{\tilde{m}}$. Will this person be expected subsequently to recover toward his original target wealth?
- Suppose a "stressed" person who is at the target level of wealth for a stressed person suddenly gets hit by a positive shock (inheritance; lottery winnings) that push his wealth above $\bar{\tilde{m}}$. Will this person be expected to recover toward his original target wealth?
- A recent study examined the consequences of giving poor people in a developing country a large lump sum of money. While they were better off for a while, eventually they subsided back into poverty indistinguishable from their initial circumstances. Is this what you would expect from the Mullainathan and Shafir (2013) model?

3. The Interest Elasticity of Saving in the Tractable Buffer Stock Model. Perfe\textsuperscript{c}sightCRRA shows that the saving rate in the perfect foresight consumption model without human capital is:

$$s_t = \left( \frac{1 - \rho^{-1}(r - \hat{\gamma})}{1 + r_{t-1}} \right).$$  \hspace{1cm} (3)

a) Explain why the response of saving to interest rates must asymptote to a negative number as $\rho$ approaches infinity. (I'm looking for intuition and economic arguments here, not just pure math).

b) We saw in class that a version of the model that incorporates uncertainty can give quite different results, especially as $a_t \downarrow 0$. Explain why saving becomes insensitive to interest rates at low levels of $a_t$. 

3
References


PART III:

Consider the textbook Solow growth model. Let $s$ be the saving rate, $n$ the population growth rate, and $d$ the depreciation rate. Assume the aggregate production function is Cobb-Douglas: $Y = L^aK^{1-a}$, where $Y$ is output, $L$ is the size of the labor force, and $K$ is the capital stock. Note there is no technological change.

A. Assume that labor and capital are both paid their marginal products. Derive expressions for the steady-state levels of (i) the real wage, (ii) the real interest rate (that is, the return on capital), and (iii) the level of pure economic profit. These expressions should be functions of the model’s parameters. Show your reasoning.

B. Taking the other parameters as given, derive an expression for the saving rate $s$ that maximizes the level of consumption in steady state. Show your reasoning.

C. Suppose the economy starts in steady state. At time $\tau$ there is a one-time shock to the economy that pushes it out of steady state. Draw graphs showing the paths of (i) total output and (ii) output per capita. The graphs should have time on the horizontal axis, and they should show what happens from the initial steady state until the economy is back in steady state after $\tau$. Briefly explain your reasoning (formal derivations are not necessary). Do this exercise for the following shocks:

- At time $\tau$, the population growth rate doubles. After $\tau$, the population growth rate stays constant at its new level.

- At time $\tau$, an epidemic causes half the labor force to die suddenly. After $\tau$, population growth resumes at the same rate as before $\tau$.

Now modify the model by allowing technological change. The production function is $Y = AL^aK^{1-a}$, where $A$ can change over time.

D. What was Solow’s assumption about the path of $A$ over time? Is this assumption realistic? In particular, does it fit the United States since World War II? Explain.

E. How have advocates of "endogenous growth theory" criticized the Solow model? How do endogenous growth models avoid the weaknesses of the Solow model? Discuss.
F. Does the Solow model help us understand why some countries have higher income per capita than others? Discuss the model’s strengths and weaknesses in this regard.

G. Does the Solow model predict that income per capita in different countries will converge over time? If this happens under certain conditions, then state the conditions. How well does the empirical evidence on convergence fit the model? Explain.