JOHNS HOPKINS UNIVERSITY
Department of Economics
Microeconomics Comprehensive Examination
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Answer all questions. The weight of each question in the final grade is given in parenthesis. You have four hours. Good luck.

Part I

I.1 (0.10) Let $C$ be a finite set of outcomes, $\mathcal{L}$ be the set of all lotteries on $C$, and $\succeq$ be a complete and transitive preference relation on $\mathcal{L}$.

(a) Define independence axiom.

(b) Show that if $\succeq$ has expected utility representation, then the independence axiom holds for $\succeq$.

(c) Under what additional conditions does the converse of (b) hold? That is, under what additional condition on $\succeq$ is it true that the independence axiom implies the existence of expected utility representation. State this additional condition clearly.

(d) Consider the following lotteries:
$L_1$: $90$ for sure, $L_2$: $90$ with probability $\frac{1}{3}$ and $0$ probability $\frac{2}{3}$, $L_3$: $20$ for sure, $L_4$: $20$ with probability $\frac{1}{2}$ and $0$ probability $\frac{1}{2}$, $L_5$: $90$ with probability $\frac{1}{4}$ and $0$ probability $\frac{3}{4}$. Suppose that an individual exhibits the following ranking of these lotteries:

$L_1 \succ L_2 \succ L_3 \succ L_4 \succ L_5$.

Is the individual’s ranking consistent with the independence axiom?

I.2 (0.15) Consider two risk neutral agents ($i = 1, 2$) who each own 1 unit of an asset, and are trying to dissolve their partnership. Player $i$’s valuation of $x \geq 0$ units of the asset is $xv_i$. Each agent knows his valuation privately, and it is common knowledge that $v_1$ and $v_2$ are drawn, independently, from uniform distribution with support $[v, \bar{v}]$. The agents dissolve their partnership through the following auction: each agent submits a bid; the higher bidder (henceforth, “the buyer”) buys the other agent’s unit of the asset and pays him an amount equal to the buyer’s bid. Derive a symmetric Bayesian Nash equilibrium in which agent $i$’s bid is linear in his valuation of the asset.
1.3 (0.15) The expenditure function is a useful tool in the theory of the consumer providing, for example, insightful relationships between Hicksian and Marshallian demand. The expenditure function can also be used to "recover" a consumer's preferences in the sense that, under certain circumstances, the upper contour sets of the consumer's utility function can be reconstructed from the expenditure function. Let \( u : \mathbb{R}_+^n \to \mathbb{R} \) be quasiconcave, continuous and monotonic. (Here, monotonicity means that \( u(x) \geq u(y) \) whenever \( x \geq y \).) Let \( \alpha \) be a real number and suppose that the problem

\[
\text{minimize } p \cdot x \text{ subject to } u(x) \geq \alpha, x \in \mathbb{R}_+^n
\]

has a solution for each \( p \in \mathbb{R}_+^n \). The optimal value for this problem is denoted \( e(p, \alpha) \) and defines the expenditure function \( p \mapsto e(p, \alpha) \). Prove that

\[
\{ x \in \mathbb{R}_+^n | u(x) \geq \alpha \} = \{ x \in \mathbb{R}_+^n | p \cdot x \geq e(p, \alpha) \text{ for all } p \in \mathbb{R}_+^n \}.
\]

[Hint: Minkowski's Theorem may be useful.]

1.4 (0.15) Consider a generalized game (i.e., an "abstract economy") with \( n \) players in the sense of Debreu defined by (i) strategy spaces \( S_i \), (ii) payoff functions \( u_i : S_1 \times \cdots \times S_n \to \mathbb{R} \) and feasible action correspondences \( \varphi_i : S_{-i} \to S_i \). Under the appropriate continuity, convexity and compactness assumptions, we showed that the generalized game has an equilibrium using Berge's Theorem and Kakutani's Fixed Point Theorem. In this problem, we will establish the existence of an equilibrium in the generalized game under different assumptions that often arise in economic applications (e.g., oligopoly theory). Suppose that for each \( i \),

(a) \( S_i = \mathbb{R}^{m_i} \) for some positive integer \( m_i \).

(b) \( u_i : \mathbb{R}^{m_1} \times \cdots \times \mathbb{R}^{m_n} \to \mathbb{R} \) has the following properties: for each \( x_{-i} \in S_{-i} \), the function \( x_i \mapsto u_i(x_{-i}, x_i) \) is differentiable and concave.

(c) \( \varphi_i : S_{-i} \to S_i \) is nonempty valued and is defined as

\[
\varphi_i(x_{-i}) = \{ x_i \in \mathbb{R}_+^{m_i} | g_i(x_{-i}, x_i) \geq 0 \} \text{ for each } x_{-i} \in S_{-i}
\]

where \( g_i : \mathbb{R}^{m_1} \times \cdots \times \mathbb{R}^{m_n} \to \mathbb{R} \) has the following properties: for each \( x_{-i} \in S_{-i} \), the function \( x_i \mapsto g_i(x_{-i}, x_i) \) is differentiable and concave.
Now suppose that \( \bar{x} = (\bar{x}_1, ..., \bar{x}_n) \in \mathbb{R}_+^{m_1} \times \cdots \times \mathbb{R}_+^{m_n} \) and \((\bar{\lambda}_1, ..., \bar{\lambda}_n) \in \mathbb{R}_+^n\) satisfy the following conditions for each \(i\):

\[
\begin{align*}
- \left[ \nabla_{x_i} u_i(\bar{x}) + \bar{\lambda}_i \nabla_{x_i} g_i(\bar{x}) \right] & \in \mathbb{R}_+^{m_i} \\
\left[ \nabla_{x_i} u_i(\bar{x}) + \bar{\lambda}_i \nabla_{x_i} g_i(\bar{x}) \right] \cdot \bar{x}_i & = 0 \\
g_i(\bar{x}) & \geq 0 \\
\bar{\lambda}_i g_i(\bar{x}) & = 0
\end{align*}
\]

Show that \( \bar{x} = (\bar{x}_1, ..., \bar{x}_n) \) is an equilibrium of the generalized game.

Part II

II.1 (0.15) On May 5th, 2009 the Financial Times reported the following story:

US lawmakers have forged a deal on a "cash-for-clunkers" car scrappage scheme that will offer taxpayer-funded vouchers of $3,500-$4,500 on an old vehicle traded in for a more fuel-efficient new one. The scheme will run for one year, and provides for about 1m new car or truck purchases, equal to slightly more than 10 per cent of annualized sales in the first four months of this year. The incentives will apply to foreign as well as domestic vehicles, no matter where they are built. The rebate will apply if the old cars achieves less than 18 miles per gallon and the new one is rated at a minimum of 22 miles per gallon. The maximum rebate will apply if the fuel efficiency of the new car is at least 10 miles per gallon higher than the old one.

President Obama has thrown his support behind the scheme as a way of helping to revive the domestic motor industry, especially the three Detroit-based carmakers, General Motors, Ford Motor and Chrysler.

(a) Analyze the proposed scheme and evaluate its impact on the US car industry and the environment.

(b) How does this scheme compare to imposing a tax on gas in terms of its welfare implications and economic stimulus?

II.3 (0.15) One maximizes welfare with the taxes one has, not the taxes one wishes one had. In particular, it would be nice to tax auto congestion but it turns out all one can tax is fuel consumption. What should one do - could doing nothing ever be the best idea? Provide an algebraic model to illustrate your analysis.
II.4 (0.15) A monopsonist buys grain from farmers that it resells in an overseas market that only it can access. The price it pays farmers is lower than this overseas price after taking into account its marketing costs. But the monopsonist also provides the farmers with fertilizer at a lower price than the world price. Is the monopsonist’s behavior, giving with one hand what he takes with the other, reasonable? Provide an algebraic model to illustrate your analysis.