JOHNS HOPKINS UNIVERSITY
Department of Economics
Microeconomics Comprehensive Examination
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Answer all questions. The weight of each question in the final grade is given in parenthesis. You have four hours. Good luck.

Part I

1. (0.20) Many economic and social games are contests where the players expend effort to increase their probability of winning a given prize. Examples include inter-firm or international R&D rivalry for a profitable innovation and bribery to secure a lucrative license or contract from a government. Contest games capture such interactions. Consider the following stylized contest game.

There are two players competing for a prize worth $1. Each player $i$ chooses an effort level $x_i \in \mathbb{R}$. When the efforts levels are $(x_1, x_2)$, the probability that player 1 wins the prize is given by $p(x_1, x_2) = \frac{2x_1}{2x_1 + x_2}$ and the probability that player 2 wins the prize is given by $1 - p(x_1, x_2)$. Player $i$'s cost of exerting effort $x_i$ is equal to $x_i$.

(a) Suppose the players simultaneously choose their effort levels.

i. Define a pure strategy equilibrium for this game.
ii. Find equilibria.
iii. The favorite is defined as the player whose chance of victory exceeds one-half in the simultaneous move game; an underdog's chance is less than one-half. Which player is the favorite?

(b) Suppose now the favorite chooses his effort level before the underdog chooses his effort level.

i. Define a pure strategy equilibrium for this sequential move game.
ii. Find equilibria.
(c) Suppose instead the underdog chooses his effort level before the favorite chooses his effort level.
   
   i. Define a pure strategy equilibrium for this sequential move game.
   
   ii. Find equilibria.

(d) Suppose the favorite can choose to move first or second. Which one will he choose?

(e) Suppose the underdog can choose to move first or second. Which one will he choose?

2. (0.13) Consider the following game:

```
   1
  /|
 A/B
  |
  /|
  3,1 L R
     |
     |
     1,1

   2
  /|
 C/D
  |
  /|
  R L
     |
     |
     -1,1
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(a) Define an equilibrium for this game.

(b) Characterize the equilibria.

Part II

3. (0.12) Each of the following statements is true or false. If true, provide a proof; and if false, a counterexample. A well-labeled diagram can serve as a counterexample, and clear English sentences can go far in delineating a proof.

(a) The set of competitive equilibria of an exchange economy in which each consumer has a strictly convex preferences is a singleton.
(b) Every competitive allocation exhibits the equal treatment property whereby identical agents obtain an identical commodity bundle.

(c) Every core allocation of a finite exchange economy can be sustained as a competitive equilibrium.

(d) Every Pareto optimal allocation of a finite exchange economy is a competitive allocation.

(e) Any upper semicontinuous correspondence \( \phi \) from a nonempty, convex, compact subset \( A \) of \( \mathbb{R}^n \) to itself has a fixed point.

4. (0.12) Each of the following statements is either valid or invalid: state your view and provide a justification for it.

(a) The convexity hypothesis on an individual production set as a formalization of technology rules out both increasing returns to scale and increasing marginal rates of substitution.

(b) A production plan \( y \) on the boundary of a closed convex production set \( Y \) need not be efficient and yet can be sustained as a profit-maximizing plan.

(c) Slater’s form of the constraint qualification in the theory of nonlinear programming is a strengthened form of the convexity of the constraint set.

(d) Basic results of the two-sector model of general economic equilibrium are simple re-workings of the neoclassical theory of the consumer.

(e) The duality and equilibrium theorems of linear programming are straightforward corollaries of the separating hyperplane theorem.

5. (0.10) Robinson and Friday have equal amounts of milk and juice to divide between themselves. Each has the same utility function given by \( u(m, j) = \max\{m, j\} \), where \( m \) denotes the amount of milk and \( j \) the amount of juice.

(a) Sketch an Edgeworth box for Robinson and Friday under some arbitrary distribution of property rights. Specify clearly the indifference curves of each, and give an interpretation of their preferences.
(b) Show the locus of Pareto optimal allocations. How does it change with the distribution of property rights?

(c) Under the assumption that Robinson "owns" all of the milk and Friday all of the juice, what is the locus of the core allocations? How does it change if the title to the two resources is equal?

(d) Is there a competitive equilibrium under each of the two systems of property rights in (3) above? Justify your answer in each case.

(e) What are the specific features of your answer in part (3) above, that are not shared by the set of competitive equilibria of a more general two-agent two-commodity exchange economy.

Part III

6. (0.23) Consider the two mechanisms describe below, designed to elicit the certainty equivalents of decision makers of a lottery, \( \ell \), depicted by a uniform distribution on the interval \([a, b]\).

**Mechanism I** - According to this mechanism a decision maker (DM) declares a number, \( x \), in the message space \([a, b]\), and a number, \( r \), is drawn from a uniform distribution on \([a, b]\). The payoffs to the DM are as follows. The DM is awarded the lottery \( \ell \) if \( r \leq x \), and he is awarded \( r \) if \( r > x \).

**Mechanism II** - According to this auction-type mechanism the DM participate in bidding against a dummy to buy the lottery \( \ell \). The price of the lottery increases continuously starting at \( a \). (For the sake of concreteness, suppose the DM and the dummy each has a button. The price is indicated by a dial that moves up continuously when both buttons are pressed and stops as soon as one of them is release). The dummy is instructed to drop out of the bidding when the price reaches \( r \), where \( r \) is a random draw from a uniform distribution on \([a, b]\). If the DM stays in when the dummy drops out he is awarded the lottery and must pay the price \( r \). If he drops out first, the dummy is awarded the lottery and the DM receives and pays nothing.

(a) Does Mechanism I implement the objective of the design (i.e., is \( x \) the certainty equivalent of \( \ell \))? Prove your assertion.

(b) Is the price at which the DM plans to drop out of the bidding in Mechanism II the certainty equivalent of the lottery? Prove your assertion.
(c) In Mechanism II, does it make a difference whether \( r \) is private information of the dummy or public information? Explain.

7. (0.10) Consider a moral hazard problem in which there are two actions, \( a \) and \( a' \) and two levels of outputs, \( x \) and \( x' \), where \( x' > x \). Assume that \( a' \) is more costly to the agent than \( a \) and the that the distribution on the set of outputs conditional on \( a' \) dominates that conditional on \( a \) according to first-order stochastic dominance. Suppose that a risk-neutral principal would like the risk-averse agent to implement the action \( a' \), and the utility of the agent's outside option is \( \bar{u} \).

(a) Formulate the principal’s problem and show that both the individual rationality constraint and the incentive compatibility constraint are binding.

(b) Does the monotone likelihood property hold in this case? Prove your assertion.

(c) Does agent compensation under the incentive contract depend on the size of \( x \) and \( x' \)? Prove your assertion.