Instructions

- You have 240 minutes for the exam plus 10 additional minutes to read it.
- Please answer all questions. The amount of time you need to answer each question is given in parenthesis. The questions are weighted by the amount of time you need to answer them.
- PLEASE START EACH QUESTION ON A NEW PAGE.
- PLEASE MAKE SURE TO DEFINE YOUR TERMS PRECISELY INCLUDING ANY NOTATION THAT YOU INTRODUCE

Part A

1. (40 minutes)
Write short succinct notes that bring out the difference between the two concepts in each of the following items. You may use verbal or symbolic or diagrammatic language but make sure that your sentences are grammatically correct.

(a) A bounded set in a metric space as opposed to a bounded set in an ordered space. Show how the definitional difference collapses for the set of real numbers.

(b) Profit-maximizing and technologically efficient production plans. What are the circumstances under which they are related?

(c) Compensated (Hicksian) and uncompensated (Marshallian) consumer demands. Is there any connection between them?

(d) The Bellman-Blackwell value function and the policy correspondence in dynamic programming. Specify the role, if any, that Berge's maximum theorem plays in relating the two.

(e) The duality and equilibrium theorems of linear programming. Specify the general result from which the two follow as consequences.

(f) A core and an individually rational allocation of a finite exchange economy. Does the "equal treatment property" have any meaning and relevance for either solution concept?
2. (40 minutes)

Debreu's book, *Theory of Value*, has an existence and a characterization claim regarding competitive allocations. Give succinct statements of each claim. The remainder of this question then asks you about the proofs regarding each of these claims.

(i) Explain the difference between an economy and a normal-form game. Give a definition of a Nash equilibrium of a game, and show how a theorem on the existence of a Nash equilibrium in a normal form game can be used to prove the existence of a competitive equilibrium of a pure exchange economy. Can one apply the theorem, as proved by Nash, directly, or does one need to modify it in some way? If a modification is required, indicate what it is.

(ii) Explain how the proof of the existence of a competitive equilibrium differs from your answer in (i) above? Sketch the basic steps of such a proof, and give a statement of the lemma on which it is based—a statement of the so-called Gale-Nikaido-Debreu lemma. Briefly explicate the meaning of each technical term used in the lemma.

(iii) It is well-understood that the lemma cannot be directly applied to prove the existence theorem in Debreu's book, and there is a need to have a recourse to some additional constructions? Indicate the reason for this need, and the constructions that address it?

(iv) Debreu used Kakutani's theorem to prove the Gale-Nikaido-Debreu lemma. Did Gale use the same proof technique? How about Nikaido? Do these alternative proofs have any substantive relevance for economic theory or they are purely of mathematical interest? Give reasons for your answer.

(v) In their formalization of a competitive economy, Arrow-Debreu assumed a finite number of producers each with a convex production set. In his formalization of a competitive economy, Lionel McKenzie assumed a single producer with a convex cone as his or her production set. Explain how an existence result pertaining to the former is technically more general than one pertaining to the latter? However, is there a substantive reason for McKenzie's formalization? Give reasons for your answer.

Part B

3. (40 minutes)

Consider the following two-stage game. In the first stage, player 1 decides how to divide $2 between herself and player 2. (Only divisions in integers are allowed.) In the second stage, they play the following simultaneous-move game with the payoffs shown in the matrix below. Player 1 chooses the row and player 2 chooses the column.
\[
L \quad M \quad R \\
U \begin{pmatrix} x, x & 0, 0 & -2, -2 \\ D \begin{pmatrix} 0, 0 & 0, 0 & 3, 3 \end{pmatrix} \end{pmatrix},
\]

Each player's payoff in the two-stage game is the sum of the dollar amount she receives in the first stage and the payoff she receives in the second stage. Consider only pure strategies in your answers below.

First, suppose that player 2 does NOT observe player 1's choice in the first stage.

(a) Draw the extensive form of the two-stage game and describe the strategy sets for the players.

(b) Does there exist a Nash equilibrium in which player 1 gives $2 to player 2 in the first stage? What about subgame perfect equilibrium? Explain.

For the remaining questions, suppose that player 2 observes player 1's choice in the first stage.

(c) Draw the extensive form of this two-stage game and describe the strategy sets for the players.

(d) For what values of $x$ does the game have a unique subgame perfect equilibrium? Explain.

(e) For what value of $x$ does the game have a subgame perfect equilibrium in which player 1 gives both dollar to player 2 in the first stage of the game? Explain.

(f) Does there exist a subgame perfect equilibrium in which player 1's payoff is less than $2? Explain.

(g) True or false: for any value of $x$, there exists a Nash equilibrium in which player 1 gives $2 to player 2 in the first stage in this game. If true, construct such a Nash equilibrium. If false, explain why.

4. (40 minutes)

Consider the following model of pre-trial negotiation between a plaintiff and a defendant. If the case goes to trial, the defendant pays the plaintiff the amount $d > 0$ in damages. It is commonly known that $d = \ell$ with probability $1/2$ and $d = h > \ell$ with probability $1/2$, but only the defendant knows the true value of $d$. Going to trial costs the plaintiff $c > 0$. For simplicity, assume that going to trial costs the defendant $0$.

The game proceeds as follows. The plaintiff first makes a settlement offer, $s > 0$. The defendant either accepts or rejects the offer. If the defendant
accepts the offer, the game ends and the plaintiff's payoff is $s$ and the
defendant’s payoff is $-s$. If the defendant rejects the offer, the plaintiff
then decides whether to go to trial. If he decides not to go to trial, the
game ends and each player receives a payoff of 0. If he decides to go
to trial, the plaintiff’s payoff is $d - c$ and the defendant’s payoff is $-d$.
Consider only pure strategies in your answers below.

(a) Describe the strategy sets of the players in this game.

(b) Suppose $c < \ell$. Does there exist a Perfect Bayesian Equilibrium
(PBE) in which the defendant accepts the settlement if and only if
the damage is high, that is, $d = h$? If so, describe such a PBE; if
not, explain why not and describe a PBE that exists.

(c) Suppose $\ell < c < (\ell + h)/2$. Does there exist a PBE in which the
defendant accepts the settlement if and only if $d = h$? If so, describe
such a PBE; if not, explain why not and find a PBE that exists.

(d) Suppose $(\ell + h)/2 < c$. Does there exist a PBE in which the defendant
accepts the settlement if and only if $d = h$? If so, describe such a
PBE; if not, explain why not and find a PBE that exists.

Part C

5. (40 minutes)

Consider an expected utility maximizing, risk-averse, decision maker (DM)
whose initial wealth is $w$ facing the following risk: With probability $p/2$
she will lose $x$, with probability $p/2$ she will lose $x + y$, $y > 0$, and with
probability $(1 - p)$ she will sustain no loss. Assume that the risk of losing
$x$ is uninsurable, but the risk of losing the extra $y$ is insurable.

(a) Suppose that DM is offered fair insurance against the risk of losing
$y$. Will she take out full insurance (that is, will the net indemnity
equal to the loss $y$)? Prove your assertion.

(b) Suppose that there is a second expected utility maximizing DM who
is more risk averse than the first in the sense of Arrow-Pratt. Facing
the same risks (described above) and the same insurance terms, will
the more risk-averse DM take out more comprehensive coverage than
the less risk-averse DM? Prove your assertion.

6. (40 minutes)

Consider a risk-averse expected utility maximizing decision maker (DM)
whose initial wealth is $w$ facing the risk of loss of $x$, with probability
$\theta$, where $w > x > 0$. Suppose that the only source of insurance is an
expected-profit maximizing monopolist.

(a) Characterize the contract the monopolist will offer to the DM if $\theta$ is
observed by the monopolist (that is, if $\theta$ is public information).
(b) Suppose, instead, that $\theta$ can take one of two values, $\{\theta_L, \theta_H\}$, where $\theta_H > \theta_L > 0$. Moreover, assume that the value of $\theta$ is private information of the DM, but the probability of $\theta_L$ is $\lambda$, $1 > \lambda > 0$, which is known to the monopoly insurer. Characterize the contract the monopolist will offer to the DM.