Noncommutative Geometry and Arithmetic.

Johns Hopkins University, March 22-25, 2011

JAMI Conference - Spring 2011

Conference Schedule

TUESDAY MARCH 22, 2011

8:00-9:00 a.m.	**Registration and Breakfast**	**Registration and Breakfast**
9:00-10:00 a.m.	A. Connes (IHES)	An overview on the Jami Conference topics
10:00-10:30 a.m.	**Coffee Break**	**Coffe Break**
10:30-11:30 a.m.	O. Viro (SUNY)	Patchworking, tropical geometry and hyperfields
11:45-12:45 p.m.	P. Lescot (U. de Rouen)	The spectrum of a characteristic one algebra
	Lunch Break	**Lunch Break**
2:30-3:30 p.m.	C. Popescu (UCSD)	Classical and Equivariant Iwasawa Theory
3:30-4:15 p.m.	**Refreshments**	**Refreschments**
4:15-5:15 p.m.	G. Litvinov (Indep. U. of Moscow)	Dequantization and mathematics over tropical and idempotent arithmetics

WEDNESDAY MARCH 23, 2011

8:00-9:00 a.m.	**Breakfast**	**Breakfast**
9:00-10:00 a.m.	O. Lorsheid (CUNY)	Blueprints and K-theory over $\mathbb{F}_1$
10:15-11:15 a.m.	C. Consani (JHU)	On the arithmetic of the BC-system I.
11:15-11:45 a.m.	**Coffee Break**	**Coffee Break**
11:45-12:45 p.m.	A. Connes (IHES)	On the arithmetic of the BC-system II.
Lunch Break	**Lunch Break**	**Lunch Break**
2:30-3:30 p.m.	Y. André (ENS Paris)	Gevrey series and arithmetic Gevrey series. A survey.
3:30-4:15 p.m.	**Refreshments**	**Refreschments**
4:15-5:15 p.m.	K. Thas (U. Gent)	Incidental geometries over $\mathbb{F}_1$

THURSDAY MARCH 24, 2011

8:00-9:00 a.m.	**Breakfast**	**Breakfast**
9:00-10:00 a.m.	K. Kato (U. of Chicago)	On conjectures of Sharifi
10:15-11:15 a.m.	J. B. Bost (U. Paris XI)	Infinite rank hermitian vector bundles
11:15-11:45 a.m.	**Coffee Break**	**Coffee Break**
11:45-12:45 p.m.	M. Vaquié (U. de Toulouse)	A new approach on non commutative motives
Lunch Break	**Lunch Break**	**Lunch Break**
2:30-3:30 p.m.	J. Lopez-Peña (U. College, London)	Crystals, clusters and combinatorial aspects of $\mathbb{F}_1$ geometries
3:30-4:15 p.m.	**Refreshments**	**Refreschments**
4:15-5:15 p.m.	E. Leichtnam (U. P.&M. Curie, Paris)	Some open problems motivated by Deninger's program.

FRIDAY MARCH 25, 2011

8:00-9:00 a.m.	**Breakfast**	**Breakfast**
9:00-10:00 a.m.	J. Borger (ANU)	Periodic Witt vectors and class field theory
10:15-11:15 a.m.	K. Kedlaya (UCSD)	The p-adic arithmetic curve: algebraic and analytic aspects
11:15-11:45 a.m.	**Coffee Break**	**Coffee Break**
11:45-12:45 p.m.	H. Ochiai (Kyushu U.)	Zeros of Witten zeta functions and absolute limits
Lunch Break	**Lunch Break**	**Lunch Break**
2:15-3:15 p.m.	B. Yalkinoglu (U. P.&M. Curie, Paris)	On Bost-Connes systems and Endomotives
3:15-3:45 p.m.	**Refreshments**	**Refreschments**
3:45-4:45 p.m.	A. Salch (JHU)	Grothendieck duality for affine M_0-schemes

ABSTRACTS OF THE TALKS

J. Borger Periodic Witt vectors and class field theory.

Abstract.

J.B.Bost Infinite rank hermitian vector bundles.

Abstract. Hermitian vector bundles over arithmetic curves are generalizations of euclidean lattices, and play a key role in Arakelov geometry. In this talk I will discuss the relevance for arithmetic geometry of some infinite rank versions of hermitian vector bundles, which, roughly speaking, may be described as Hilbert spaces equipped with integral lattices.

A. Connes On the arithmetic of the BC-system II.

Abstract. This is the second part of the talk presented by K. Consani. In this talk we show that the role of the Riemann zeta function, as partition function of the BC-system over $\mathbb{C}$ is replaced, in the p-adic case, by the p-adic L-functions and the polylogarithms whose values at roots of unity encode the KMS states. We use the Iwasawa theory to extend the KMS theory to a covering of the completion $\mathbb{C}_p$ of an algebraic closure of $\mathbb{Q}_p$. We show that our joint previous work on the hyperring structure of the adèle class space, combines with p-adic analysis to refine the space of valuations on the cyclotomic extension of $\mathbb{Q}$ as a noncommutative space intimately related to the integral BC-system and whose arithmetic geometry comes close to fulfill the expectations of the "arithmetic site". Finally, we explain how the integral BC-system appears naturally also in the construction of de Smit and Lenstra of the standard model of $\bar{\mathbb{F}}_p$ which singles out the subsystem associated to the $\hat{\mathbb{Z}}$-extension of $\mathbb{Q}$.

K. Consani On the arithmetic of the BC-system I.

Abstract. The talk will present some of the latest developments on the study of the arithmetic of the BC-system obtained in collaboration with A. Connes. In particular, it will be shown that for each prime p and each embedding σ of the multiplicative group of an algebraic closure of $\mathbb{F}_p$ as complex roots of unity, there exists a p-adic, indecomposable representation of the integral BC-system as additive endomorphisms of the big Witt ring of $\bar{\mathbb{F}}_p$. These are the p-adic analogues of the complex, extremal KMS_∞-states of the BC-system.

K. Kedlaya The p-adic arithmetic curve: algebraic and analytic aspects.

Abstract. Very recently, Fargues and Fontaine have given a new description of p-adic Hodge theory based on the properties of a certain one-dimensional scheme, in which the basic results can be formulated and proved in the language of vector bundles on algebraic curves. Moreover, this scheme naturally relates to certain Berkovich nonarchimedean analytic spaces whose geometry is extremely suggestive; for instance, the p-adic Witt vector Frobenius naturally appears as a deck transformation. This is related to the fact that from the point of view of nonar-

chimedean analytic geometry, passing from a perfect ring of characteristic p to its ring of Witt vectors behaves like adjoining a polynomial variable. I'll describe these developments and give some hints about where this subject might be going next.

K. Kato On conjectures of Sharifi.

Abstract. Romyar Sharifi formulated remarkable conjectures on the relations between modular curves and ideal class groups of cyclotomic fields. Takako Fukaya and I obtained partial results on his conjectures. I explain the results and the methods.

E. Leichtnam Some open problems motivated by Deninger's program.

Abstract. Deninger's program suggests that trace formulae for arithmetic zeta functions might be interpreted as Lefschetz trace formulae for certain unknown foliated spaces. We shall explain how this is a motivation to state tractable open problems.

P. Lescot The spectrum of a characteristic one algebra.

Abstract. Let $\mathbb{B}_1 = \{0, 1\}$ with $1 + 1 = 1$ denote the smallest characteristic one semifield. In two previous papers I have studied the fundamental properties of $\mathbb{B}_1$–algebras.

It turns out that, given a $\mathbb{B}_1$–algebra A, a certain set of ideals of A (the saturated spectrum $Prs(A)$) can be equipped with a Zariski-type topology. We shall discuss the fundamental topological properties of such a space.

These results suggest a possible definition for "$\mathbb{B}_1$-schemes".

G. Litvinov Dequantization and mathematics over tropical and idempotent arithmetics.

Abstract. Tropical mathematics can be treated as a result of a dequantization of the traditional mathematics as the Planck constant tends to zero taking imaginary values. This kind of dequantization is known as the Maslov dequantization and it leads to a mathematics over tropical algebras like the max-plus algebra. The so-called idempotent dequantization is a generalization of the Maslov dequantization. The idempotent dequantization leads to idempotent mathematics over idempotent semirings. For example, the field of real or complex numbers can be treated as a quantum object whereas idempotent semirings can be examined as "classical" or "semiclassical" objects (a semiring is called idempotent if the semiring addition is idempotent, i.e. $x \oplus x = x$).

In the spirit of N.Bohr's correspondence principle there is a (heuristic) correspondence between important, useful, and interesting constructions and results over fields and similar results over idempotent semirings. A systematic application of this correspondence principle leads to a variety of theoretical and applied results.

J. Lopez-Peña Crystals, clusters and combinatorial aspects of $\mathbb{F}_1$ geometries.

Abtsract. One of the first motivating examples of varieties over the field with one element was the one of Chevalley groups (split reductive group schemes). Chevalley groups have been realized as defined over $\mathbb{F}_1$ in several different approaches. In the seek of a purely intrinsic

definition of such an $\mathbb{F}_1$ geometry we showed as a proof of concept how Lusztig construction of the integral form of a split reductive group scheme by means of the canonical basis could be thought of as the "extension of scalars" from $\mathbb{F}_1$ to the integers in some (still elusive) intrinsic notion of $\mathbb{F}_1$ geometry based on crystals. In this talk we shall proceed further down those lines and show how the theory of cluster algebras presents itself as a good candidate to encode the $\mathbb{F}_1$-skeleton of not only Chevalley groups, but also other algebraic varieties that have a combinatorial nature.

O. Lorscheid Blueprints and K-theory over $\mathbb{F}_1$.

Abtsract. In this talk, we introduce a new category of algebraic objects, called blueprints, that contains commutative monoids and commutative semi-rings as full subcategories. A scheme theory associated these objects reproduces usual schemes as well as $\mathcal{M}_0$-schemes as full subcategories.

Beside the extremal cases of schemes and $\mathcal{M}_0$-schemes, the generalized geometry of blueprints contains new interesting objects, such as "improved" cyclotomic field extensions $\mathbb{F}_{1^n}$ of $\mathbb{F}_1$, "archimedean valuation rings" and algebraic groups over $\mathbb{F}_1$. It also combines well with Frederic Paugam's viewpoint on analytic geometry (in the sense of Huber and Berkovich), which results in an analytic geometry over $\mathbb{F}_1$. This analytic geometry over $\mathbb{F}_1$ seems promising to make the long assumed connection between tropical geometry and $\mathbb{F}_1$-geometry precise.

Finally we review K-theory over $\mathbb{F}_1$, as developed jointly with Chenghao Chu and Rekha Santhanam, in this more general context.

H. Ochiai Zeros of Witten zeta functions and absolute limits.

Abstract. The Witten zeta function is constructed from a compact group as the Dirichlet series over the unitary dual. In this talk we report on a study on zeros of Witten zeta functions of compact groups. We show the existence of certain integral zeros. We investigate "Witten L functions" also. In some special cases we show the simplicity of these zeros. Moreover we calculate absolute limits for some Witten zeta functions. This would indicate the existence of $\mathbf{Z}_1$ consiting of one adic integers.

A. Salch Grothendieck duality for affine M_0-schemes.

Abstract. M_0-schemes, scheme-like objects which look Zariski-locally like commutative monoids with zero element, are one model for geometry over the field with one element. We investigate Grothendieck duality for the morphism from Spec$\mathbb{Z}$ to SpecM_0. Since Grothendieck duality is a construction in the derived category of a scheme, we provide a natural generalization of the notion of the derived category of a scheme which applies to M_0-schemes, and we show that the bounded-below derived category of SpecM_0 is equivalent, as a triangulated category, to the homotopy category of bounded-below spectra studied in stable homotopy theory.

Let f denote the map from Spec$\mathbb{Z}$ to SpecM_0. We describe the functors Rf_*, Lf^*, and the Grothendieck duality functor $f^!$ defined on the derived categories of Spec$\mathbb{Z}$ and $SpecM_0$, and we compute the homology groups of the dualizing complex $f^!(M_0)$, which are nontrivial in

degree 1 and vanish in all other degrees. We show that this nontrivial homology group has an interesting topological description, as the group of phantom maps from the suspension of the integral Eilenberg-Maclane spectrum to the sphere spectrum, and it has an interesting number-theoretic description, as a Shimura-variety-like adelic quotient associated to the additive group scheme.

K. Thas Incidental geometries over $\mathbb{F}_1$.

Abstract. In a paper which was published in 1957, Tits made a seminal and provocative remark which alluded to the fact that through a certain analogy between the groups $\mathbf{GL}_n(q)$ (or $\mathbf{PGL}_n(q)$), q any prime power, and the symmetric groups $\mathbf{S}_n$ on n symbols, one should interpret $\mathbf{S}_n$ as a Chevalley group "over the field of characteristic one", $\mathbb{F}_1$. Of course this field does not really exist, but there should be a deep theory of geometries which behave as if they were defined over $\mathbb{F}_1$. In that same paper, Tits described the (spherical) buildings over $\mathbb{F}_1$ as the apartments arising in the spherical buildings of the same type (and rank), and the corresponding Chevalley groups then become the Weyl groups of the associated BN-pair. (Also in rank 2, this approach works well in a more synthetic setting.)

Much later, several groups of people introduced various theories of Algebraic Geometry over $\mathbb{F}_1$ (aiming, for one, at a proof of the Riemann Hypothesis).

In my talk, I plan to review some of these theories. I will then speak about the concept of *incidence geometries over* $\mathbb{F}_1$, and look at the interrelations with its Algebraic Geometry counterpart. If time permits, Chevalley groups (and more general automorphism groups of combinatorial objects) will come into play.

O. Viro Patchworking, tropical geometry and hyperfields.

Abstract.

B. Yalkinoglu On Bost-Connes systems and Endomotives.

Abstract. An open problem in the theory of Bost-Connes systems over arbitrary number fields is to construct so called arithmetic subalgebras. The aim of our talk is to show that the framework of Endomotives, introduced by A. Connes, K. Consani and M. Marcolli, and a classification result of Λ-rings due to J. Borger and B. de Smit provide the necessary tools to solve the problem of constructing arithmetic subalgebras in full generality.

If time permits we will discuss further applications of our construction.